SOME REPRODUCING KERNELS FOR THE UNIT DISK

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Introduction. Let $S(t)$ denote the class of functions φ analytic in the unit disk U with center 0 and satisfying

$$(1) \qquad \int_U\int |\varphi(z)|(1-|z|^2)^t\,dxdy < \infty \quad (z = x+iy)$$

for t real. In this paper we shall prove that for λ and ν properly restricted, $|\zeta| < 1$ and $\varphi \in S(t)$, the following formulas are valid:

$$(2) \qquad \varphi(\zeta) = \frac{(\lambda+1)^\nu}{\Gamma(\nu)\pi}\int_U\int \frac{\varphi(z)(1-|z|^2)^\lambda}{(1-\bar{z}\zeta)^{\lambda+2}}\, ln^{\nu-1}\left(\frac{1-\bar{z}\zeta}{1-|z|^2}\right)dx\,dy\ ,$$

and

$$(3) \quad \varphi^{(m)}(\zeta) = \frac{\lambda+1}{\pi}\iint \bar{z}^m \frac{\varphi(z)(1-|z|^2)^\lambda}{(1-\bar{z}\zeta)^{\lambda+2+m}}\sum_{i=0}^{m} a_i ln^{\nu-1-i}\left(\frac{1-\bar{z}\zeta}{1-|z|^2}\right)dx\,dy\ ,$$

where the a_i are suitably chosen constants (with respect to φ and the variables z and ζ). Finally, if

$$(4) \qquad \begin{aligned} F_n(\zeta,\nu,\lambda) = \frac{(-1)^{n+1}}{\pi}&\iint \frac{\varphi(z)(1-|z|^2)^\lambda}{\bar{z}^n(1-\bar{z}\zeta)^{\lambda+2-n}} \\ &\cdot\left[\frac{(\lambda+1)^{\nu-1}}{\Gamma(\nu+n-1)}\, ln^{\nu+n-2}\left(\frac{1-\bar{z}\zeta}{1-|z|^2}\right)\right. \\ &\left.+\frac{1}{\Gamma(n)}\, ln^{n-1}\left(\frac{1-\bar{z}\zeta}{1-|z|^2}\right)\right]dxdy\ , \end{aligned}$$

then $F_n(\zeta,\nu,\lambda)$ has the property that

$$(5) \qquad \frac{d^n}{d\zeta^n}F_n(\zeta,\nu,\lambda) = \varphi(\zeta)\ .$$

Formula (2) reduces to the well known results of Ahlfors [1] and Bergman [2] for particular choices of the parameters t, λ, and ν. The author is indebted to Professor Ahlfors for suggesting this problem.

Notation. Define

$$N(z,\lambda) = (1-|z|^2)^\lambda\ ,$$
$$D(z,\zeta,\lambda) = (1-\bar{z}\zeta)^\lambda\ ,$$
$$L(z,\zeta,\nu) = ln^{\nu-1}\left(\frac{1-\bar{z}\zeta}{1-|z|^2}\right)$$

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