EXTREME EIGEN VALUES OF TOEPLITZ FORMS ASSOCIATED WITH JACOBI POLYNOMIALS

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Introduction. Let $t(\theta)$ be a real function in $L^1(T)$ where T is the real numbers modulo 1, and let

$$c(k) = \int_T t(\theta)e^{-2\pi ik\theta}d\theta \qquad k = 0, 1, \cdots ,$$

$$C_n = [c(j-k)]_{j,k=0,\cdots,n} .$$

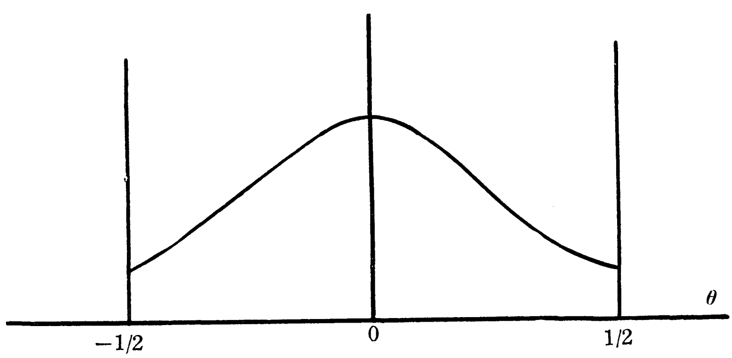
C_n is the Toeplitz matrix of index n associated with $t(\theta)$. C_n is clearly Hermitian and thus has real eigen values,

$$\lambda_{n,1} \geqq \lambda_{n,2} \geqq \cdots \geqq \lambda_{n,n+1} .$$

For some time studies have been made of the asymptotic behaviour of these eigen values as $n \to \infty$. Thus, for example, if $N(a, b; n)$ is, for n fixed, the number of $\lambda_{n,k}$'s which satisfy $a \leqq \lambda_{n,k} \leqq b$, and if $\nu(y)$ is the Lebesgue measure of the set $\{\theta \mid t(\theta) < y\}$ then

$$(1) \qquad \lim_{n\to\infty} n^{-1}N(a, b; n) = \nu(a) - \nu(b) ,$$

provided a and b are points of continuity of ν. This result was proved by Szegö, see [2; p. 64]. Detailed investigations have also been made of the behaviour of $\lambda_{n,k}$ as $n \to \infty$ while k is fixed, under various additional assumptions on $t(\theta)$. Suppose that $t(\theta)$ is continuous for $\theta \in T$, has a unique absolute maximum at $\theta = 0$, and that $t(\theta)$ is twice continuously differentiable in a neighborhood of $\theta = 0$ with $t''(0) < 0$.



It was shown in 1953 by Kac, Murdock, and Szegö that under these assumptions

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