A NOTE ON PSEUDO-CREATIVE SETS AND CYLINDERS

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1. Notation and Definitions. We will use N to denote the set of all nonnegative integers. Unless specifically mentioned otherwise, all sets are considered subsets of N. If A is a set, $A' = N - A$. Since we consider only sets of nonnegative integers, we will not use Cartesian products of sets but will instead work with images of Cartesian products under some effective mapping. More specifically, if A and B are sets, let $A \otimes B = \{(a, b) \mid a \in A \text{ and } b \in B\}$. Let τ be any one-to-one effective mapping of $N \otimes N$ onto N. Then we define $A \times B$ to be $\tau(A \otimes B)$, and we abbreviate $\tau((a, b))$ to $\langle a, b\rangle$. (This is the notation introduced by Rogers in [4].) Given integers a and b we can always effectively find the integer $\langle a, b\rangle$, and given the integer $\langle a, b\rangle$ we can always effectively find a and b.

In [2], Myhill has called a set a cylinder if it is recursively isomorphic to $B \times N$ for some r.e. set B; however we will follow Rogers in calling a set, A, a cylinder if it is recursively isomorphic to $B \times N$ for any set B. Such a set A is called a cylinder of B.

For definitions of recursive, simple, and creative sets, see [3]. A noncreative, recursively enumerable (r.e.), set A has been called pseudo-creative if for every r.e. set $B \subset A'$ there is an infinite r.e. set $C \subset A'$ such that $B \cap C = \emptyset$. A nonrecursive r.e. set A has been called pseudo-simple if there is an infinite r.e. set $B \subset A'$ such that $A \cup B$ is simple. We will denote the class of all recursive sets by $\mathscr{C}_0$, the class of all simple sets by $\mathscr{C}_1$, the class of all pseudo-simple sets by $\mathscr{C}_2$, the class of all pseudo-creative sets by $\mathscr{C}_3$, and the class of all creative sets by $\mathscr{C}_4$. These classes are pairwise disjoint and every r.e. set falls into one of the classes ([2]).

Let A and B be sets. We write $A \leqq_1 B$ if there is a one-to-one recursive function such that $x \in A$ if and only if $f(x) \in B$, $A \leqq_m B$ if there is some recursive function g such that $x \in A$ if and only if $g(x) \in B$, and $A \leqq_{btt} B$ if A is reducible to B via bounded truth-tables. If there is no recursive function g such that $x \in A$ if and only if $g(x) \in B$, we write $A \nleqq_m B$. If both $A \leqq_m B$ and $B \leqq_m A$, we write $A \equiv_m B$.

2. Introduction and preliminaries. In [2] it is shown that the class of pseudo-creative sets is nonempty by proving that the cylinder of any nonrecursive, noncreative, r.e. set is pseudo-creative. In this

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