ON SOME FINITE GROUPS AND THEIR COHOMOLOGY

KUNG-WEI YANG

The purposes of this paper are: (I) to characterize the finite groups whose 2-Sylow subgroups are not isomorphic to a generalized quaternion group and which have periodic cohomology of period 4, (II) to show that all possible cohomologies of such a group G can be realized by direct sums of G-modules which belong to a specific finite family of G-modules.

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The reader is referred to [1, Ch. XII] for basic notions, definitions and notations concerning cohomology of finite groups. The only departure from [1, Ch. XII] is the following: we shall say that a finite group G has periodic cohomology of period k if k is the *least* positive integer such that $\hat{H}^k(G, Z)$ contains a maximal generator [1, pp. 260–261]. And to avoid typographical difficulties we will denote by $Z(l)$ the cyclic group of order l.

PROPOSITION I. *Let G be a finite group whose 2-Sylow subgroups are not isomorphic to a generalized quaternion group. Then G has periodic cohomology of period 4 if and only if G has a presentation*

$$G = \{\sigma, \tau \colon \sigma^s = 1, \tau^t = 1, \tau\sigma\tau^{-1} = \sigma^{-1}\}, \text{ with the conditions}$$

(i) *s is an odd integer >1,*
(ii) *t is a positive even integer prime to s.*

Proof. Let G be a finite group whose 2-Sylow subgroups are not isomorphic to a generalized quaternion group and which has periodic cohomology of period 4. It is well-known [1, Theorem 11.6, p. 262] that if a finite group has periodic cohomology (of finite period) every Sylow subgroup of the group is either cyclic or is a generalized quaternion group. Since we assume that the 2-Sylow subgroups of G are not isomorphic to a generalized quaternion group, every Sylow subgroup of G is cyclic. It is also well-known [6, Theorem 11, p. 175] that a finite group G containing only cyclic Sylow subgroups is metacyclic and has a presentation

$$G = \{\sigma, \tau \colon \sigma^s = 1, \tau^t = 1, \tau\sigma\tau^{-1} = \sigma^r\}, \text{ with the conditions}$$

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