TRANSFORMATIONS OF DOMAINS IN THE PLANE AND APPLICATIONS IN THE THEORY OF FUNCTIONS

MOSHE MARCUS

In this paper we shall consider a family of transformations S_n $(n = 1, 2, \cdots)$ operating on open or closed sets in the complex plane z. S_n is defined relatively to a fixed point called the center of transformation, and it transforms an open set into a starlike domain which, for $n > 1$, is also n-fold symmetric with respect to this point. Therefore, for $n > 1$, S_n may be classified as a method of symmetrization. This method of symmetrization was already defined by Szegö [4] for domains which are starlike with respect to the center of transformation.

The definition of S_n will be extended (in the way usually used for symmetrizations) so that S_n will operate also on a certain class of functions and a family of condensers, in the plane. It will be proved that S_n diminishes the capacity of a condenser and this result will be used in order to obtain certain theorems in the theory of functions.

1. Definitions and notations.

The transformations S_n are defined as follows.

DEFINITION 1. *Let Ω be an open set in the plane z, which does not contain the point at infinity, and let z_0 be a point of Ω. If $|z - z_0| < \rho$, $(0 < \rho)$, is a circle contained in Ω, we define:*

$$L_\rho(\varphi) = \int_E \frac{dr}{r}\,, \tag{1}$$

where $|z - z_0| = r$ and

$$E = \{z \mid z \in \Omega, |z - z_0| > \rho, \arg(z - z_0) = \varphi\}\,;$$

$$L_\rho^{(n)}(\varphi) = \frac{1}{n}\sum_{k=0}^{n-1} L_\rho\Big(\varphi + \frac{2\pi k}{n}\Big)\,; \tag{2}$$

$$\begin{cases} R(\varphi) = \rho \exp\{L_\rho(\varphi)\} \\ R^{(n)}(\varphi) = \Big[\prod_{k=0}^{n-1} R\Big(\varphi + \frac{2\pi k}{n}\Big)\Big]^{1/n} = \rho\exp\{L_\rho^{(n)}(\varphi)\}\,. \end{cases} \tag{3}$$

Evidently, $R^{(n)}(\varphi)$ does not depend on ρ.

Received March 28, 1963. This paper represents part of a thesis submitted to the Senate of the Technion, Israel Institute of Technology, in partial fulfillment of the requirements for the degree of Master of Science. The author wishes to thank Professor E. Netanyahu for his guidance and help in the preparation of this work.