A SUFFICIENT CONDITION THAT AN ARC IN S^n BE CELLULAR

P. H. DOYLE

An arc A in S^n, the n-sphere, is cellular if $S^n - A$ is topologically E^n, euclidean n-space. A sufficient condition for the cellularity of an arc in E^3 is given in [4] in terms of the property local peripheral unknottedness (L.P.U) [5]. We consider a weaker property and show that an arc in S^n with this property is cellular.

If A is an arc in S^n we say that A is p-shrinkable if A has an end point q and in each open set U containing q in S^n, there is a closed n-cell $C \subset U$ such that q lies in Int C (the interior of C), while BdC (the boundary of C) meets A in exactly one point. We note that A is p-shrinkable is precisely the condition that A be L.P.U. at an endpoint [5]. There is, however, a good geometric reason for using the p-shrinkable terminology here; the letter p denotes pseudo-isotopy.

LEMMA 0. *Let C^n be a closed n-cell and D^n a closed n-cell which lies in int C^n except for a single point q which lies on the boundary of each n-cell. If there is a homeomorphism h of C^n onto a geometric n-simplex such that $h(D^n)$ is also an n-simplex, then there is a pseudo-isotopy ρ_t of C^n onto C^n which is the identity on BdC^n, while $\rho_1(D^n)$, the terminal image of D^n, is the point q.*

The proof of this is omitted since it depends only on the same result when C^n and D^n are simplices.

LEMMA 1. *Let C^n be a closed n-cell and B an arc which lies in int C^n except for an endpoint b of B on BdC^n. Then there is a pseudo-isotopy of C^n onto C^n which is fixed on BdC^n and which carries B to b.*

Proof. Since $B \cap BdC^n = b$ we note that there is in C^n an n-cell D^n which contains B in its interior except for the point b, $D^n - b \subset \text{Int}\, C^n$, and D^n is embedded in C^n as in Lemma 0. Thus Lemma 0 can be applied to shrink B in the manner required by the Lemma.

THEOREM 1. *Let A be an arc in S^n such that for each subarc B of A, B is p-shrinkable. Then every arc in A is cellular.*

Proof. The proof is by contradiction. If A contains a non-cellular

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