IRREDUCIBLE GROUPS OF AUTOMORPHISMS OF ABELIAN GROUPS

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The group Γ of automorphisms of the abelian group A is termed irreducible, if 0 and A are the only Γ-admissible subgroups of A. It is our aim to investigate the influence of the structure of the abstract group Γ upon the structure of the pair A, Γ. In this respect we succeed in proving the following results:

If Γ is locally finite, then A is an elementary abelian p-group and the centralizer Δ of Γ within the ring of endomorphisms of A is a commutative, absolutely algebraic field of characteristic p. If we impose the stronger hypothesis that Γ possesses an abelian torsion subgroup of finite index, then the rank of [the vector space] A over Δ is finite and Γ is a group of finite rank. If we add the further hypothesis that the orders of the elements in Γ are bounded, then A and Γ are finite.

NOTATIONS

Locally finite group	= group whose finitely generated subgroups are finite.
Almost abelian group	= group possessing abelian subgroups of finite index
Group of finite rank	= group whose finitely generated subgroups may be generated by fewer than a fixed number of elements
m-group	= group by whose subgroups the minimum condition is satisfied.

Composition of the elements in the basic abelian group A is denoted by addition. The effect of the endomorphism σ of A upon the element a in A will usually be denoted by $a\sigma$ unless A is considered as a vector space over some field of scalars in which case the scalars may appear to the left of the vectors.

PROPOSITION. *If the irreducible group Γ of automorphisms of the abelian group $A\,[\neq 0]$ is locally finite, then*

(a) *the centralizer Δ of Γ [within the ring of endomorphisms of A] is a commutative, absolutely algebraic field of characteristic p, a prime,*

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