ON THE STRUCTURE OF INFRAPOLYNOMIALS WITH PRESCRIBED COEFFICIENTS

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Introduction. The main result of this paper is Theorem 5 which deals with the structure of infrapolynomials with prescribed coefficients. This theorem was quoted (without proof) in a previous paper [Shisha and Walsh, 1961][1], and was used there to prove a few results concerning the geometrical location of the zeros of some infrapolynomials with prescribed coefficients [loc. cit., Theorems 11, 12, 16, 17]. Two similar results are given here in Theorem 6.

We refer the reader to the Introduction of the last mentioned paper for a review of the development of the concept of infrapolynomial. Here we shall just mention two of the underlying definitions.

A. Let n and q be natural numbers ($q \leq n$), $n_1, n_2, \cdots, n_q$ integers such that $0 \leq n_1 < n_2 \cdots < n_q \leq n$, and S a set in the complex plane[2]. An nth *infrapolynomial on* S *with respect to* $(n_1, n_2, \cdots, n_q)$ is a polynomial $A(z) \equiv \sum_{\nu=0}^{n} a_\nu z^\nu$ such that no $B(z) \equiv \sum_{\nu=0}^{n} b_\nu z^\nu$ exists, satisfying the following properties.

(1) $B(z) \not\equiv A(z)$,

(2) $b_{n_\nu} = a_{n_\nu}$ $(\nu = 1, 2, \cdots, q)$,

(3) $|B(z)| < |A(z)|$ whenever $z \in S$ and $A(z) \neq 0$, and

(4) $B(z) = 0$ whenever $z \in S$ and $A(z) = 0$.

B. Let n be a natural number. *A simple n-sequence* is a sequence having one of the forms

$$(0, 1, \cdots, k, n-l, n-l+1, \cdots, n)\ [k \geq 0,\ l \geq 0,\ k+l+2 \leq n]\,,$$
$$(0, 1, \cdots, k)\ [0 \leq k < n],\ (n-l, n-l+1, \cdots, n)\ [0 \leq l < n]\,.$$

Theorem 5 may yield information on the location of the zeros of an nth infrapolynomial $A(z)$ on a set S with respect to a simple n-sequence σ. For it allows (under quite general conditions) to set $A(z) \equiv B(z)\, D(z)$ where $D(z)$ is a polynomial all of whose zeros lie in S, whereas $B(z)$ is a divisor of a polynomial $Q(z)$ whose structure is given by the theorem. By studying the location of the zeros of $Q(z)$, one may get information on the location of the zeros of $A(z)$. By this method, Theorems 11, 12, 16, 17 [loc. cit.] were proved. (Compare

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[1] Dates in square brackets refer to the bibliography.

[2] We deal throughout this paper with the *open* plane of complex numbers.