CONTRIBUTIONS TO BOOLEAN GEOMETRY OF p-RINGS

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1. Introduction. In a paper in this journal [7], J. L. Zemmer proposed two problems relating to the geometry of the Boolean metric space of a p-ring. (A p-ring is a ring R in which $px = 0$ and $x^p = x$ for some positive prime p, and all $x \in R$. The axioms of a p-ring imply its commutativity.) The first problem asked for necessary and sufficient conditions in order that a subset of such a space (hereafter called a p-space) be a metric basis; the second problem was the determination of congruence indices for p-spaces, with respect to the class of Boolean metric spaces. The present paper contains solutions to these questions as well as a brief discussion of certain properties of the group of motions of a p-space, and an introduction to analytic geometry in a p-space. The reader is referred to Zemmer's paper for definitions not contained herein.

2. Metric bases for p-spaces. Let us recall the following definition.

DEFINITION 2.1. A subset S of a Boolean metric space M is called a *metric basis*, if and only if x, y in M and $d(x, s) = d(y, s)$ for all $s \in S$ imply $x = y$.

Let R be a p-space and B its Boolean ring of idempotents. It is well known that B is a subdirect sum of $GF(2)$ [6]. Denote by B^* the complete direct sum of these same rings.

Associate with every subset S of R a subset $\bar{S}$ of B^* defined as follows:

Let $S_{j,k}$ be the subring of B^* consisting of those elements z of B^* having the property

$$z \subseteqq \bigcap_{s \in S} (s - j)^{p-1}(s - k)^{p-1}$$

for $j, k = 0, 1, 2, \cdots, p - 1,\ j \neq k$.

Let

$$\bar{S} = \bigcup_{set} S_{j,k}[j < k;\ j, k = 0, 1, 2, \cdots, p - 1] .$$

THEOREM 2.1. *Let R be a p-space with Boolean ring of idempotents B. If S is a subset of R then S is a metric basis for R if*

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