INVERSION AND REPRESENTATION THEOREMS FOR A GENERALIZED LAPLACE TRANSFORM

J. M. C. JOSHI

1. Introduction. In a series of recent papers I have discussed various properties and inversion theorems etc. for the transform

$$F(x) = \frac{\Gamma(\beta+\eta+1)}{\Gamma(\alpha+\beta+\eta+1}\int_0^\infty (xy)^\beta {}_1F_1(\beta+\eta+1; \alpha+\beta+\eta+1; -xy)f(y)dy \, . \tag{1.1}$$

where $f(y) \in L0, \infty)$, $\beta \geqq 0$, $\eta > 0$.

$$= A\int_0^\infty (xy)^\beta \psi(x, y)f(y)dy$$

where for convenience we denote $\Gamma(\beta+\eta+1)/\Gamma(\alpha+\beta+\eta+1)$ by A and ${}_1F_1(a; b; -xy)$ by $\psi(xy)$; a and b standing respectively for $\beta+\eta+1$ and $a+\alpha$. For $\alpha=\beta=0$ (1.1) reduces to the wellknown Laplace transform

$$F(x) = \int_0^\infty e^{-xy} f(y)dy \, . \tag{1.2}$$

The transform (1.1), which may be called a generalization of the Laplace transform, arises if we apply Kober's operators of fractional integration [2] to the function $x^\beta e^{-x}$[1].

The object of the present paper is to obtain an inversion and a representation theorem for the transform (1.1) by using properties of Kober's operators defined below.

2. Definition of operations. The operators given by Kober are defined as follows.

$$I^+_{\eta,\alpha}[f(x)] = \frac{1}{\Gamma(\alpha)} x^{-\eta-\alpha}\int_0^x (x-u)^{\alpha-1}u^\eta f(u)du$$

$$K^-_{\zeta\,\alpha}[f(x)] = \frac{1}{\Gamma(\alpha)} x^\zeta \int_n^\infty (u-x)^{\alpha-1}u^{-\zeta-\alpha}f(u)du$$

where $f(x) \in L_p(0, \infty)$, $1/p + 1/q = 1$, if $1 < p < \infty$ and $1/p$ or $1/q$ 0 if p or q=1, $\alpha > 0$, $\zeta > -(1/p)$, $\eta > -(1/q)$.

The Mellin transform $\bar{M}f(x)$ of a function $f(x) \in L_p(0, \infty)$ is defined as

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