ON A GENERALIZED STIELTJES TRANSFORM

J. M. C. Joshi

1. Introduction. In a series of recent papers [1–4] I have discussed various properties and inversion theorems etc. for the transform

$$(1.1)\qquad F(x) = \frac{\Gamma(\beta+\eta+1)}{\Gamma(\alpha+\beta+\eta+1)} \times \int_0^\infty (xy)^\beta {}_1F_1(\beta+\eta+1;\ \alpha+\beta+\eta+1;\ -xy) f(y)dy$$

where $f(y) \in L(0, \infty)$, $\beta \geqq 0$, $\eta > 0$.

$$F(x) = A\int_0^\infty (xy)^\beta F(x, y) f(y)dy$$

where, for convenience, we denote $\Gamma(\beta+\eta+1)/\Gamma(\alpha+\beta+\eta+1)$ by A and ${}_1F_1(a; b; -xy)$ by $F(x, y)$, a and b standing respectively for $\beta+\eta+1$ and $\alpha+a$. For $\alpha=\beta=0$ (1.1) reduces to the well known Laplace Transform

$$(1.2)\qquad F(x) = \int_0^\infty e^{-xy} f(y)dy\ .$$

The transform (1.1), which may be called a generalization of Laplace Transform, arises when we apply Kober's [5] operators of Fractional Integration [6] to $x^\beta e^{-x}$.

The object of the present paper is to give a generalization of Stieltjes Transform, to give an inversion theorem for it and to use that inversion theorem to obtain an inversion theorem for the transform (1.1). In another paper (to appear elsewhere) I have found out inversion operators directly for (1.1).

2. Generalized Stieltjes transform. We prove

THEOREM 2.1. *If*

$$(2.1)\qquad \phi(s) = \int_0^\infty e^{-sx} F(x)dx$$

where $F(x)$ *is given by the convergent integral* (1.1), *then*

$$(2.2)\qquad \phi(s) = \frac{A\Gamma(\beta+1)}{s}\int_0^\infty \left(\frac{y}{s}\right)^\beta F\left(a, \beta+1; b; -\frac{y}{s}\right) f(y)dy$$

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