MINIMUM PROBLEMS OF PLATEAU TYPE IN THE BERGMAN METRIC SPACE

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Dedicated to my teacher Professor C. Loewner on his seventieth birthday

1. Introduction. In this paper we are concerned with the existence of minimal surfaces with respect to the B-area (see § 4) and related problems in a bounded domain D in the space C^2 of two complex variables z_1, z_2.

Let $K_D(z, \bar{z})$, $z = (z_1, \cdots, z_n)$, be the Bergman kernel function of a bounded domain D in the space C^n of n complex variables. Throughout this paper, we assume $K_D(z, \bar{z})$ has the boundary value infinity at every point on the boundary of D. The kernel $K_D(z, \bar{z})$ enables us to define the Bergman metric

$$ds_D^2(z) = \sum_{\mu,\nu=1}^{n} T_{\mu\bar{\nu}}(z, \bar{z}) dz_\mu d\bar{z}_\nu, \; T_{\mu\bar{\nu}} = \frac{\partial^2 \log K_D}{\partial z_\mu \partial \bar{z}_\nu}, \tag{1.1}$$

which is invariant with respect to pseudo-conformal mappings [4, pp. 51–53]. Using (1.1) we construct (see § 2) the complete Bergman metric space (D, d) over D and state a theorem for complete Riemannian spaces that for any two points in D, there exists a minimal curve with respect to d which connects the two points.

In § 3 we show that, if D is a plane domain bounded by finitely many boundary components $b_1, b_2, \cdots, b_n$, then there exists a minimal closed curve with respect to d among those curves which are homotopic to a fixed inner boundary component, say b_1, in $\overline{D(b_1)}$ (see § 3 for notation). If D is doubly connected, there exists a unique minimal closed curve in D. Furthermore, we prove a distortion theorem which gives bounds for the Bergman lengths of the minimal closed curves.

Analogous results are obtained in the case of two complex variables replacing the length by the B-area.

For a closed Jordan curve Γ in a complete metric space (D, d), we ask whether there exists a minimal surface with respect to the B-area which spans Γ. Answers to this question which constitute the main result of this paper are given in § 4.

As a generalization of § 3, we consider a domain D which is topologically equivalent to a product domain of the form $D_1 \times D_2$,

Received July 5, 1963. This paper represents the essential contents of the author's doctor thesis at Stanford University. The author wishes to express his thanks to Professor S. Bergman and Professor C. Loewner for their pertinent criticism in the preparation of this paper. This research was supported by Navy Contract Nonr-225(11) and NSF Grant 21344.