MAPPINGS OF BOUNDED CHARACTERISTIC INTO ARBITRARY RIEMANN SURFACES

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Introduction. Throughout this paper we consider analytic mappings $f(z)$ of an arbitrary open Riemann surface R into an arbitrary Riemann surface S. Heins [3] introduced the class of Lindelöfian maps when R is hyperbolic, and defined them in terms of Green's functions; further contributions have been made by Rao [4], [5]. In the case of planar regions these maps are the classical functions of bounded characteristic.

Sario [6], [7], has utilized principal functions [1] on the range surface to obtain generalizations of the main theorems for mappings of R into S. In this paper a different first main theorem is obtained in which the proximity function is a generalization of Nevanlinna's proximity function by means of the substitution of a principal function for the logarithmic function. It is shown that the resulting class of functions of bounded characteristic are the Lindelöfian maps, and that an extremal decomposition characterization of these functions can be obtained as in the classical case.

1. An auxiliary family of functions. Analytic mappings from an arbitrary open surface R into an arbitrary surface S can be considered in terms of families $\mathscr{T}$ of LH functions, i.e., harmonic functions, with isolated logarithmic singularities having integral coefficients. For the purposes of this paper we slightly generalize the term, parametric disk: $\Delta = (Q, \mu)$ is a parametric disk if Q is a classical parametric disk, and there is defined on it a metric μ that is a real scalar multiple of the induced metric.

We let ζ be the local variable on S, and fix $\sigma \in S$ and a parametric disk at σ. If S is closed we define $t(\zeta, \sigma, \alpha)$ for $\alpha \in S\backslash\sigma$ (set difference) as the LH function on S which has singularities $\log|\zeta - \alpha|$ and $-\log|\zeta - \sigma|$ and is normalised by

$$\lim_{\zeta\to\sigma} (t(\zeta, \sigma, \alpha) + \log|\zeta - \sigma|) = 0$$

in terms of the fixed parametric disk. At α a parametric disk is fixed such that

$$\lim_{\zeta\to\alpha} (t(\zeta, \sigma, \alpha) - \log|\zeta - \alpha|) = 0$$

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