ON THE REFLECTION OF HARMONIC FUNCTIONS AND OF SOLUTIONS OF THE WAVE EQUATION

VLADIMIR FILIPPENKO

Introduction. While the analytic extension of a harmonic function across analytic differential boundary conditions is always possible for the case of two independent variables [3], no comparable global theorem exists for harmonic functions in $N > 2$ variables.

This work is concerned with the problem of global extension of a harmonic function $U(x, y, z)$ across a plane on which U satisfies a linear differential boundary condition of the form

$$B(U) \equiv \frac{\partial U}{\delta z} + P_n(x, y)U = 0 \qquad \text{on } \sigma(z = 0) ,$$

where $P_n(x, y)$ is a polynomial of degree n. It is assumed here that the given function U is C^1 in the closure of a cylindrical domain $R: \{x^2 + y^2 < \rho^2, \ -l < z < 0\}$.

The possibility of harmonic reflection is obvious for $n = 0$, $P_n =$ const. as $B(U)$ itself is harmonic. Since it vanishes on $z = 0$, it can be extended harmonically, and the harmonic extension of U can then be found by integrating with respect to z. But such procedure is no longer available in our case. We shall show, how our problem can be reduced to that of solving an initial value problem of a certain hyperbolic differential equation (1.22) of order $2n$ with distinct characteristic surfaces (of normal type).

Classical considerations yield the analyticity of U on σ and, therefore, its harmonic extensibility across σ into a neighborhood of σ. Our result asserts that this neighborhood is the whole of the mirror image of R, denoted by $\bar{R}$.

Our method consists of constructing a new function $V(x, y, z)$ from U and a differential expression in V (see (1.6) and (1.18)), which is harmonic in R and vanishes on $z = 0$. Thus, this expression in V can be first extended into $R \cup \sigma \cup \bar{R}$ as a harmonic function $\varphi(x, y, z)$. The solution of the differential equation thus obtained for V in $\bar{R}$ is impeded by its degeneracy. To remove this degeneracy we add to the differential equation the Laplacian of V and its higher derivatives in such a way as to obtain a normal hyperbolic problem (1.22), whose solution is guaranteed by a result of I. G. Petrovsky. This modification of the differential equation can be done in infinitely many ways, in particular, so as to make the characteristic surfaces close down on

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