ANOTHER CHARACTERIZATION OF THE n-SPHERE AND RELATED RESULTS

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In [5] we defined an irreducible $B(J)$-cartesian membrane and an excluded middle membrane property EM, and used these to characterize the n-sphere. There the class $B(J)$ was of $(n-1)$-spheres contained in a compact metric space S. Since part of the proof does not depend upon the fact that elements of $B(J)$ are $(n-1)$-spheres, we consider the possibility of other entries in the class $B(J)$. Recent developments in this direction have been made by Bing in [2] and by Andrews and Curtis in [1]. In [3] and [4] Bing constructed a space B not homeomorphic with E^3, which has been called the dogbone space. By Theorem 6 of [2], the sum of two cones over the one point compactification $\bar{B}$ of B is homeomorphic with S^4. This sum of two cones over a common base X is called the suspension of X.

In [1] Andrews and Curtis showed that if α is a wild arc in S^n that the decomposition space S^n/α is not homeomorphic with S^n. They proved, however, that the suspension of S^n/α is always homeomorphic with S^{n+1} for any arc $\alpha \subset S^n$. The reader will easily see that a class $\bar{B}$ or of S^n/α as described will satisfy the conditions for a class $B(J)$ for which an n-sphere will have property EM.

The results below were obtained in considering such spaces, and Theorem 1 below is a weaker characterization of the n-sphere than is Theorem 2 of [5]. We find it difficult to determine the properties $J \in B(J)$ must have for S to have Property EM, as is shown by our Theorem 4 below.

I. Definition and basic properties. Let S always be a compact metric space and let $B(J)$ be a class of mutually homeomorphic subcontinua of S. We put conditions on this general class $B(J)$ in our theorems below.

We define a $B(J)$-cartesian membrane as we did in [5] and [6]. Let F be a compact subset of S containing $J \in B(J)$. Let M be a subcontinuum of F, $b \in M$ and C be homeomorphic to J. Denote by $(C \times M, b)$ the decomposition space [10: pp 273–274] of the upper semi-continuous decomposition of the cartesian product $C \times M$, where the only nondegenerate element is taken to be $C \times b$ (intuitively the decomposition space is a sort of generalized cone with vertex at the point $C \times b$). With this notation we give:

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