COVERING SPACES OF PARACOMPACT SPACES

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Introduction. Let $\tilde{X}$ and X be two Hausdorff spaces and f a continuous[1] mapping of $\tilde{X}$ into X. We say that f is a covering mapping if f maps $\tilde{X}$ onto X and there exist an open covering[1] $\mathscr{V}$ of X having the following property:

(1) For every $V \in \mathscr{V}$, $f^{-1}[V]$ is a union of a family $\mathscr{F}(V)$ consisting of pairwise-disjoint open sets each of which is mapped homeomorphically onto V by f.

The pair $(\tilde{X}, f)$ is called a covering space of X.

If X is a metric space, nothing can be said, in general, about the diameters of the elements of the covering $\mathscr{V}$ of X, the diameters of the elements of $\mathscr{F}(V)$, $V \in \mathscr{V}$, or any isometric properties of f, as can be seen from the following example:

EXAMPLE 1. Let $\tilde{X}$ be the real line with the usual metric, X the unit circle $|z| = 1$ in the complex Z-plane with length of minor arc as the distance between two points and finally f the function: $f(\tilde{x}) = e^{i|\tilde{x}|\tilde{x}}$.

Then $(\tilde{X}, f)$ is a covering space of X, if $\mathscr{V}$ is the set of arcs of length one. Now, let V be the unit spherical region (i.e. the arc of length one) with $z = 1$ as centre. One can easily see that $f^{-1}[V]$ consists of intervals of the form $2\kappa\pi - 1 < x^2 < 2\kappa\pi + 1$ and the infinum of their diameters is zero. Thus if $\tilde{V} \in \mathscr{F}(V)$, $f \mid \tilde{V}$ has in general no isometric properties. But it is easily seen that the metric in $\tilde{X}$ can be changed (without changing the topology of $\tilde{X}$) in such a way that $f \mid \tilde{V}$ will be an isometry for every $\tilde{V} \in \mathscr{F}(V)$ and every $V \in \mathscr{V}$. This leads to the following problem:

Problem. Let $(\tilde{X}, f)$ be a covering space of a metrisable space X. Does there exist a metric $\tilde{\rho}$ in $\tilde{X}$ and a metric ρ in X, inducing the topologies of $\tilde{X}$ and X respectively and such that the family $\mathscr{S}$ of unit spherical regions in (X, ρ) has the following property:

(A) For every $S \in \mathscr{S}$, $f^{-1}[S]$ is a union of a family $\mathscr{F}(S)$, consisting of pairwise-disjoint unit spherical regions in $(\tilde{X}, \tilde{\rho})$ each of

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[1] In this paper all mappings and functions are assumed to be continuous, and all coverings to be open. The qualifying adjectives are omitted accordingly.