COVERINGS OF ALGEBRAIC GROUPS AND LIE ALGEBRAS OF CLASSICAL TYPE

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Introduction. By a Lie algebra of classical type we shall mean a Lie algebra L over an algebraically closed field K of characteristic $p > 7$ which possesses a standard Cartan subalgebra, that is, an abelian Cartan subalgebra H such that L and H satisfy the axioms of Mills and Seligman [11].

Let G be the algebraic component of the identity in the automorphism group of L. In [7] Curtis constructs an irreducible projective representation of G from each of the irreducible restricted representations of L and a group G^* which is a covering group for G in the sense that

(i) there is a covering homomorphism mapping G^* onto G whose kernel is contained in the center of G^*;

(ii) each of the projective representations of G constructed can be lifted to an irreducible representation of G^*.

It is the purpose of this paper to investigate the structure of this covering group and to identify it. The main structure theorem presented here, after noting that G^* is an irreducible linear algebraic group, is that the decomposition of L as the direct sum of simple ideals induces a decomposition of G and G^* as the direct product of the corresponding groups for the simple ideals. The identification problem is thus reduced to the "simple case" and for Lie algebras of type A_n or C_n results have been obtained by Curtis [7]. In the last section we give a complete treatment for the orthogonal Lie algebra with respect to a quadratic form Q, that is, for types B_n and D_n. The principal result obtained is that the covering group is birationally isomorphic to the reduced Clifford group associated with Q.

These results are of some interest in view of recent work of Steinberg [13]. Working with the simple groups defined by Chevalley [5] Steinberg constructs a covering group in terms of generators and relations which is naturally isomorphic to the simply connected covering group of the simple Chevalley group. Combining Steinberg's results with ours it is not difficult to show that the two covering groups are birationally isomorphic.

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