ARENS MULTIPLICATION AND CONVOLUTION

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1. Introduction. Let L denote the group algebra of a locally compact Abelian (LCA) group $\mathscr{G}$. For elements x and y in L the product of x and y is given by

$$xy(\beta) = \int x(\beta - \alpha)y(\alpha)d\alpha \qquad \beta \in \mathscr{G} ,$$

where the integral is taken over the entire group and with respect to Haar measure.

Let L^* and L^{**} denote the first and second conjugate spaces of L, respectively. As a result of [1], a multiplication can be introduced in L^{**} in the following manner. Let $x, y \in L$; $f, g \in L^*$; and $F, G \in L^{**}$. The elements xf and $F \circ f$ in L^* and $G \circ F$ in L^{**} are defined by:

$$xf(y) = f(xy) \qquad y \in L , \tag{1.1}$$

$$F \circ f(x) = F(xf) \qquad x \in L , \tag{1.2}$$

$$G \circ F(f) = G(F \circ f) \quad f \in L^* . \tag{1.3}$$

The multiplication in L^{**} given by (1.3) will be referred to as the Arens product. Some of the properties of the Arens product in L^{**} have been developed in [2].

It is well-known that the spaces L^* and L^{**} have realizations in terms of functions on $\mathscr{G}$ [5, p. 148] and finitely additive measures on $\mathscr{G}$ [6], respectively. One difficulty which arises with the Arens product is that there seems to be no means of obtaining the functions and measures which correspond to elements of the form $F \circ f$ and $G \circ F$, respectively. To avoid excessive notation we will use $f, g, \cdots$ to denote elements of L^* and their corresponding realizations as functions. Any statement involving $f, g, \cdots$ as functions will be interpreted as a locally almost everywhere statement (see [5, p. 141]) even though a reference to locally almost everywhere (l.a.e.) may not appear. Similarly, $F, G, \cdots$ will denote elements of L^{**} and their corresponding realizations as finitely additive measures.

In the case of xf, an obvious application of the Fubini theorem yields

$$xf(\beta) = \int f(\beta + \alpha)x(\alpha)d\alpha \qquad \beta \in \mathscr{G} , \tag{1.4}$$

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