## BOUNDED GENERALIZED ANALYTIC FUNCTIONS ON THE TORUS

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1. Introduction. We shall operate in Euclidean  $k$ -space,  $E_k$ ,  $k \geq 2$ , and use the following notation:

$$
x = (x_1, \dots, x_k); \qquad y = (y_1, \dots, y_k);
$$
  
\n
$$
\alpha x + \beta y = (\alpha x_1 + \beta y_1, \dots, \alpha x_k + \beta y_k);
$$
  
\n
$$
(x, y) = x_1 y_1 + \dots + x_k y_k; \qquad |x| = (x, x)^{1/2}.
$$

T<sub>k</sub> will designate the k-dimensional torus  ${x; -\pi < x_j \leq \pi, j=$  $1, \dots, k$ , v will always designate a point a distance one from the origin, i.e.,  $|v|=1$ , and m will always designate an integral lattice point. If f is in  $L^1$  on  $T_{k_1}$  then  $\hat{f}(m)$  will designate the mth Fourier coefficient of f, i.e.,  $(2\pi)^{-k}$  |  $f(x)e^{-i(m,x)}dx$ .

*JT<sup>k</sup>* We shall say that  $f(x)$  in  $L^2$  on  $I_k$  is a generalized analytic function  $T^T$  if there exists a such that f is in  $A$ , where  $A = A^+ \cup A^+$ tion on  $I_k$  if there exists *v* such that *f* is in  $A_v$ , where  $A_v = A_v \cup A_{-v}$ , and  $A^+$  is defined as follows: and *At* is defined as follows:

*j* is in  $A_i$  if there exists an  $m_0$  such that if  $m \neq m_0$  and  $m_0 \geq 0$  than  $\hat{f}(m) = 0$ 

 $(m - m_0, v) \geq 0$ , then  $J(m) = 0$ .<br>We shall say that  $f(x)$  in I We shall say that  $f(x)$  in  $L^2$  on  $I_k$  is a strictly generalized ana<br>  $f(x)$  is  $L^2$  is the second that  $f(x)$  is  $D$  when  $\frac{1}{k}$ *λc* function on  $I_k$ <br> $-P_{k+1}P_{k}$  and if there exists a v such that  $j$  is in  $D_v$ , where  $\Delta_{\theta} = D_{\theta}$  U  $D_{-\theta}$ , and  $D_{\theta}$  is defined as follows:

<sup>+</sup> if there exists an  $m_0$  and  $a \gamma$  with  $0 < \gamma < 1$  such  $J$  is in  $D_v$  if there exists an  $m_0$ <br>if  $(m - m_0) \leq \alpha | m - m_0| + \alpha$ that if  $(m - m_0, v) \leq 7 | m - m_0|$ , then  $f(m) = 0$ .<br>It is quite clear that  $P \subset A$ . In this paper , v)  $\lt$  1 |  $m - m_0$ <br>aloon that  $P \subset$ 

It is quite clear that  $B_v \subseteq A_v$ . In this paper, we shall obtain a result which is false for bounded functions in  $A<sub>v</sub>$  but which is true for bounded functions in  $D_v$ . It is primarily with the class  $D_v$  and its<br>ortension to finite complex measures that the classical paper of Bochnor extension to finite complex measures that the classical paper of Bochner [2, p. 718] is concerned. On  $T_k$ , it is essentially with the class  $A_n$  $[2, 0, 10]$  is concerned. On  $I_k$ , it is essentially with the class  $A_v$ <br>that the papers of Helsen and Leudenslager [5] [6] and de Leeuw that the papers of Helson and Lowdenslager [5], [6], and de Leeuw and Glicksberg [4] are concerned.

We shall be concerned in this paper with a class of functions  $C<sub>v</sub>$ which for bounded functions is intermediate between the two classes  $B_n$  and  $A_n$ .  $B_v$  and  $A_v$ .

We first note that if  $f$  is in  $B_v$ , then  $\sum_m |f(m)| e^{imx} < \infty$  for  $\sigma < 0$ . For with  $||f|| = 1 < \sigma < \infty$  designating the *I*<sup>p</sup>-narm of every  $\sigma < \sigma$ . For with  $||f||_p$ ,  $1 \ge p \ge \infty$ , designating the Z<sup>1</sup>-norm of f on  $T_k$ , we see that there exists a  $\gamma$  with  $0 < \gamma < 1$  and an  $m_0$  such that that

Received October 8, 1963. This research was supported by the Air Force Office of Scientific Research.