ON COVERINGS

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1. Introduction. Recently [2, 3, 4, 5] renewed interest has been aroused in the notion of covering and related problems, originally posed by Steiner [8] and later reformulated by Moore [6] as problems of the existence of *tactical configurations.*

A tactical configuration $C(k, l, \lambda, n)$ $(n \geqq k \geqq l)$ is a set of unordered k-tuples of n different elements, such that each l-tuple of these elements appears exactly λ times.

In view of the importance of the special cases $\lambda = 1$ and $l = 2$ the notions of *tactical systems* $S(k, l, n)$ for $C(k, l, 1, n)$ *and balanced incomplete block designs* (*BIBD*) $B(k, \lambda, n)$ for $C(k, 2, \lambda, n)$ have also been used.

A necessary condition [6] for the existence of a tactical configuration $C(k, l, \lambda, n)$ is known to be

$$(1) \qquad \lambda\binom{n-h}{l-h}\Big/\binom{k-h}{l-h} = \text{integer}\,, \qquad h = 0, 1, \cdots, l-1\,.$$

For $h = 0$ this integer, namely

$$(2) \qquad \lambda\binom{n}{l}\Big/\binom{k}{l}$$

is clearly the number of elements in $C(k, l, \lambda, n)$.

Condition (1) has been proved to be sufficient for $l = 2$, $k = 3$, $\lambda = 1$ by Moore [6] and Reiss [7], for $l = 2$, $k = 3$, $\lambda = 2$ by Bose [1], for $l = 2$, $k = 3$ and $k = 4$ and every λ, for $l = 2$, $k = 5$ $\lambda = 1, 4$ and 20, and for $l = 3$, $k = 4$ and every λ by Hanani [3, 4, 5].

These results for $\lambda = 1$ show—and we note this here for future references—that necessary and sufficient conditions for the existence of tactical systems $S(4, 2, n)$, $S(5, 2, n)$ and $S(4, 3, n)$ are, respectively

$$(3) \qquad n \equiv 1 \text{ or } 4 \pmod{12}$$

$$(4) \qquad n \equiv 1 \text{ or } 5 \pmod{20}$$

$$(5) \qquad n \equiv 2 \text{ or } 4 \pmod 6$$

More general *coverings* $R(k, l, \lambda, n)$ existing for every n may be defined.

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