ON HIGH SUBGROUPS

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One of the purposes of this paper is to answer the following three questions:

(1) What groups G with $G^1 = 0$ are direct summands of all groups containing them as high subgroups?

(2) If G is a Σ-group, are all high subgroups of G endomorphic images of G (see [3] and [4])?

(3) If G is a torsion Σ-group, is every subgroup of G a Σ-group (see [3])?

The answer is affirmative to (2) and negative to (3). However an affirmative answer can be given to (3) when $|G^1| \leqq \aleph_0$.

All groups in this paper will be assumed to be additively written abelian groups. For the most part, the notation and terminology of [2] will be followed. If G is a group, G_t will denote the torsion subgroup of G and G^1 the subgroup of elements of infinite height, that is, $G^1 = \bigcap_{n=1}^{\infty} nG$. A torsion group is said to be closed if each p-primary component is a closed p-group (see [2], pp. 114–117). A mixed abelian group is said to split if it decomposes into a direct sum of a torsion and torsion free group. By the n-adic topology on the group G, we shall mean the topology defined by taking as neighborhoods of 0 the subgroups nG for each positive integer n. A subgroup H of G is said to be a high subgroup if H is maximal in G with respect to $H \cap G^1 = 0$. If H is a high subgroup of G, then H is pure in G and G/H is divisible (see [3]). If all high subgroups of G are direct sums of cyclic groups, then G is said to be a Σ-group. If one high subgroup of G is a direct sum of cyclic groups, then all high subgroups of G are isomorphic and G is a Σ-group (see [4]).

1. High subgroups. Let G be an arbitrary abelian group and let D be a minimal divisible group containing G^1. Then let K be the amalgamated sum of G and D over G^1, that is, K is the abelian group generated by the elements of G and D subject only to $G \cap D = G^1$. (K can be realized as $(G + D)/L$ where L is the subgroup of $G + D$ consisting of all elements of the form $(x, -x)$ with $x \in G^1$.) It then follows that $K/G = \{G, D\}/G \cong D/(G \cap D) = D/G^1$, and similarly that $K/D \cong G/G^1$.

LEMMA 1. *If D is minimal divisible containing G^1 and if K is the amalgamated sum of G and D over G^1, then*

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