MOSAICS OF METRIC CONTINUA AND OF QUASI-PEANS SPACES

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1. Introduction. A collection $\{(X_a, \mathscr{T}_a) : a \in A\}$ is a *mosaic of topological spaces on a set* X if and only if each $(X_a, \mathscr{T}_a)$ is a topological space; $X = \bigcup\{X_a : a \in A\}$; and the following compatibility condition is satisfied: for all $a, b \in A$ and all subsets M of X_a, if M is $\mathscr{T}_a$-closed then $M \cap X_b$ is $\mathscr{T}_b$-closed. For a mosaic of topological spaces on X, the *mosaic topology* $\mathscr{T}$ is defined as follows: for all $M \subseteqq X$, M is $\mathscr{T}$-closed if and only if $M \cap X_a$ is $\mathscr{T}_a$-closed for all $a \in A$. Clearly, each $(X_a, \mathscr{T}_a)$ is then a closed subspace of $(X, \mathscr{T})$. If each $(X_a, \mathscr{T}_a)$ is a compact metric space, a Peano space, or an arc then $(X, \mathscr{T})$ is called a *mosaic space*, a *curve space*, or an *arc space*, respectively.

Davison [1] introduced the theory of mosaics of topological spaces, concentrating on the theory of mosaic spaces and establishing some properties of curve spaces and arc spaces. Our purpose is to study mosaics of spaces which are between being compact metric and Peano; namely, compact, connected, metric spaces and compact, locally connected, metric spaces, which we shall call *metric continua* and *quasi-Peano spaces*, respectively. Mosaics of these spaces with the mosaic topology will be called *m-continuum spaces* and *quasi-curve spaces*, respectively.

It might seem quite natural to consider, also, mosaics of compact metric spaces each of which has only finitely many components. Doing so, however, yields nothing more than is obtained by studying mosaics of metric continua, as Theorem 3.3 will show.

In this paper, we give a characterization of m-continuum spaces and sufficient conditions for a mosaic spaces to be an m-continuum space. The property of m-strong connectedness is studied in connection with m-continuum spaces and a sufficient condition for an m-continuum space to be locally m-strongly connected is given. Finally, the equivalence of curve spaces and quasi-curve spaces is established.

For ease of reference we now list results of Davison to be used in this paper.

1.1. *Notation.* If S is a sequence of points, S_J denotes the point set associated with S.

1.2. THEOREM. ([2], Lemma 1.5) *Let* $(X, \mathscr{T})$ *be a mosaic space*

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