IDENTITY AND UNIQUENESS THEOREMS FOR AUTOMORPHIC FUNCTIONS

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1. Introduction. Let C and D denote the unit circle and the unit disk, respectively, and let $\rho(z, z')$ denote the non-Euclidean hyperbolic distance between the points z and z' in D [3, Chapter II]. Bagemihl and Seidel have proved the following identity theorem [2, Theorem 3, p. 13].

THEOREM A. *Let $f(z)$ be a meromorphic function of bounded characteristic in D, and let $\{z_n\}$ be a sequence of points in D with at least two limit points in C, such that $|z_n| \to 1$ and $\rho(z_n, z_{n+1}) < M$ for every n, where M is a positive constant. If $f(z_n) \to c$, then $f(z) \equiv c$.*

There is also a corresponding uniqueness theorem [2, Theorem 4, p. 14].

THEOREM B. *Let $f(z)$ and $g(z)$ be meromorphic functions of bounded characteristic in D, and let $\{z_n\}$ be a sequence of points in D with at least two limit points in C, such that $|z_n| \to 1$ and $\rho(z_n, z_{n+1}) < M$ for every n, where M is a positive constant. If $\{f(z_n) - g(z_n)\} \to 0$, then $f(z) \equiv g(n)$.*

Along the same lines, Bagemihl has proved an identity theorem for normal functions [1, Theorem 3, p. 4].

THEOREM C. *Let $f(z)$ be a normal meromorphic function and let $\{z_n\}$ be a sequence of points in D with at least two limit points in C, such that $|z_n| \to 1$ and $\rho(z_n, z_{n+1}) \to 0$. If $f(z_n) \to c$, then $f(z) \equiv c$.*

This paper will investigate such identity and uniqueness theorems for automorphic functions. It shall be shown that there is a result analoguous to Theorem C for automorphic functions with Fuchsian groups of the first kind. However, an example will show that there is no corresponding theorem for automorphic functions with Fuchsian groups of the second kind. In the case of automorphic functions with

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