OPEN IDEALS IN $C(X)$

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A group topology on the ring $C(X)$, of all real-valued, continuous functions on X, is said to have the *ideal closure property*, (I.C.P), in case the closure of any ideal is simply the intersection of all maximal ideals containing it. In this paper we consider which ideals of $C(X)$ can be open with respect to such a topology.

In § 2, a characterization of such ideals is given and it is shown that the family $\mathscr{S}$, of all such ideals, is itself a fundamental system of neighborhoods of zero with respect to a ring topology having I.C.P. In § 3 we consider the two extremes, where $\mathscr{S}$ is the family of all ideals and where $\mathscr{S}$ consists only of finite intersections of maximal ideals. The former class is characterized as the class of p-space (spaces for which every prime ideal of $C(X)$ is maximal) and the latter as the class of pseudo-compact spaces (spaces for which every $f \in C(x)$ is bounded). In the final section it is shown that if P is a countable discrete subset of the Stone-Cech compactification of X, then $\bigcap\{M^p; p \in P\} \in \mathscr{S}$ if and only if P is C-embedded in $X \cup P$.

1. The notation and terminology will be that of [1]. Many of the arguments will depend upon theorems and exercises of [1]. In order to avoid lengthy restatements of these results, when such results are used, we will simply give a reference to the appropriate statement. To simplify the reader's task all references will be given to [1]. The original source of these results can be determined by consulting the notes at the end of this book.

Throughout the paper X will denote a topological space and $C(X)$ the ring of all real-valued continuous functions on X. The term "topology", unless explicitly stated to the contrary, will always mean Hausdorff topology.

Although it is assumed that the reader is familiar with the material in the first few chapters of [1], we will recall some of the basic definitions and results which will be used throughout. For $f \in C(X)$ we set $Z(f) = \overleftarrow{f}(0)$ and for an ideal I we set $Z[I] = \{Z(f); f \in I\}$. An ideal I is called a z-ideal if $Z(g) \in Z[I]$ implies $g \in I$. It will be recalled that there is a one-to-one correspondence between the maximal ideals of $C(X)$ and the points of the Stone-Cech compactification, βX, of X. Explicitly this correspondence, $p \to M^p$, is given by the Gelfand-

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