CUT POINTS IN TOTALLY NON-SEMI-LOCALLY-CONNECTED CONTINUA

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1. Introduction. F. Burton Jones has shown [4, Theorem 15] that the set of weak cut points of a compact metric continuum that is not semi-locally-connected at any point of an open set U of M is dense in U (the proofs apply to locally peripherally compact complete metric continua). It is the purpose of this paper to extend Jones' results by establishing stronger cutting properties.

The theory is given here for locally compact metric spaces but applies, with appropriate modifications, to locally peripherally bicompact, regular spaces of the general class mentioned in [2].

2. Definitions and preliminary theorems. A point p of a continuum M is a *weak cut point* of M (or *cuts M weakly*) if there are two points x and y of $M-\{p\}$ such that each subcontinuum of M that contains both x and y contains p also. In this case p *cuts* x *from* y *weakly* in M.

A continuum M is *semi-locally-connected* at a point p of M if each open subset U of M containing p contains an open subset V of M containing p, the complement of which relative to M consists of a finite number of components. A continuum M is *totally nonsemi-locally-connected* (on a point set A) if M is not semi-locally-connected at any point (of A). A continuum M is *locally peripherally aposyndetic* at a point p of M if each open subset U of M containing p contains an open subset V of M containing p such that, for some collection $(H_1, \cdots, H_n)$ of subcontinua of M, $(\bigcap_{i=1}^n H_i) \cap (\bar{V}-V)=\phi$ and p is in the (nonvoid) interior W of $(\bigcap_{i=1}^n H_i) \cap V$, relative to M. In this case W is a *peripheral aposyndesis subset* of U and V is a set *associated with* W and U.

EXAMPLE. The Cartesian product of a cantor set and a simple closed curve, with one of the cantor sets shrunk to a point, is locally peripherally aposyndetic at each point but aposyndetic at only one point.

THEOREM 1. *A locally compact metric continuum M is locally peripherally aposyndetic on a dense G_δ subset of an open subset D*

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