ON ABSTRACT AFFINE NEAR-RINGS

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1. Introduction. We shall limit ourselves to near-rings for which addition is commutative. They will be known as abelian near-rings. We assume that the distributive law $(b + c)a = ba + ca$ holds, but the law $a(b + c) = ab + ac$ does not necessarily hold. (This is consistent with the usual convention that the product AB of two operators A and B stands for B followed by A, e.g., consider the near-ring of all mappings of a group into itself.) Our aim is to generalize the results of [1] and [2] to a class of near-rings which we call abstract affine near-rings.

2. Abelian near-rings. We first define two subsets $L(R)$ and $C(R)$ of a near-ring R. (When convenient, we call these sets L and C. $L(R)$ is the set of all elements $a \in R$ which satisfy $a(b + c) = ab + ac$ for all b and c in R. $C(R)$ is the set of all elements $a \in R$ which satisfy $ab = a$ for all b in R. Note that, in general, $0 \cdot a = 0$ and $(-a)b = -(ab)$.

PROPOSITION 1. *L is a subring of B.*

Proof. If $a, b \in L$, then

$$\begin{aligned}(a + b)(x + y) &= a(x + y) + b(x + y) = ax + ay + bx + by \\ &= (ax + bx) + (ay + by) = (a + b)x + (a + b)y ,\end{aligned}$$

hence $a + b \in L$. Since $0 \cdot a = 0$ for all $a, 0 \in L$. Also if $a \in L$, then

$$\begin{aligned}(-a)(x + y) &= -[a(x + y)] = -[ax + ay] = (-ax) + (-ay) \\ &= (-a)x + (-a)y ,\end{aligned}$$

hence $-a \in L$. Furthermore if $a, b \in L$, then $ab(x + y) = a(bx + by) = abx + aby$, hence $ab \in L$. This completes the proof. Note that if R contains an identity e, then $e \in L$.

DEFINITION. An r-ideal is a subgroup closed under multiplication on the left and right by arbitrary elements of R. An ideal I is a subgroup closed under right multiplication by elements of R and which furthermore satisfies $y(x + a) - yx \in I$ for all $a \in I$, $x \in R$, $y \in R$.

PROPOSITION 2. *C is an r-ideal of R.*

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