

IDEMPOTENT SEMIGROUPS WITH DISTRIBUTIVE RIGHT CONGRUENCE LATTICES

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A great deal of effort in the study of semigroups has been spent in an attempt to adopt group theoretic methods to semigroups and to find suitable analogues for group concepts that will be significant in the general structure theory of semigroups. Of particular importance in the study of groups are the various relationships between a group and its subgroups. As is well-known each subgroup in a group induces a decomposition of the group into right cosets. In turn, this decomposition corresponds to an equivalence relation that is invariant under right multiplication. We call such an equivalence relation a right congruence. Since there is a one-to-one correspondence between the set of right congruences of a group and the set of subgroups of the group it is clear that any subgroup-group relationship can be translated into one involving these right congruences.

In semigroup theory the importance of the subsemigroup structure to the nature of the semigroup is not quite so clear. This is due primarily to the fact that there is very little relationship between the homomorphisms of a semigroup and the subsemigroups of the semigroup. Thus in studying lattices associated with semigroups we have chosen to study the right congruences of a semigroup rather than the more obvious analogue of subgroup, the subsemigroup, studied by Ego, et al, [3, 7, 8].

In §1 we show that these right congruences form a complete lattice which is compactly generated in the sense of Crawley and Dilworth [2, p. 2]. It is natural to ask what are the implications for the semigroup of restraints which may be placed on this related lattice.

As a first problem in this area we seek a characterization of those semigroups whose lattice of right congruences is distributive. For groups this answer was determined by Ore [6, Theorem 4] to be the locally cyclic groups. It is shown in §2 that the lattice of right congruences of a locally cyclic semigroup is distributive. (It should be noted here that Severin [7] has shown that the lattice of semigroups of a locally cyclic semigroup is not necessarily distributive.) However, as is seen, not all semigroups with distributive right congruence lattices need be locally cyclic. Thus the characterization problem remains. While we have no solution to this problem in general, we do give in §§3 and 4 necessary and sufficient conditions for an idempotent semigroup to have a distributive lattice of right congruences. §3 treats