PSEUDO-FRATTINI SUBGROUPS

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1. Introduction. During the current interest in Frattini subgroups, $\Phi(G)$, of finite groups, G, two well-defined characteristic subgroups have been overlooked. They are the intersection of the normal maximal subgroups, $R(G)$, and the intersection of the self-normalizing maximal subgroups, $L(G)$; in each case one sets $R(G) = G$ or $L(G) = G$ if the respective maximal subgroups do not exist properly. This paper examines the properties of these subgroups and introduces an upper L-series which is defined as $L_0 = L(G)$, $L_1 = [L(G), G], \cdots$, $L_j = [L_{j-1}, G], \cdots$; its role being analogous to the upper central series. The terminal member of an L-chain, $L^*(G)$, is called an L-commutator. Whenever $L^*(G) = 1$, $L(G)$ coincides with the hypercenter of G. In conclusion it is shown that a group having $L^*(G) = 1$ and having all subgroups of $G/\Phi(G)$ the direct product of elementary Abelian p-group is equivalent to a group having each proper subgroup nilpotent.

It is assumed that the reader is familiar with the definitions and properties of ascending and descending central series, nilpotent groups, and Frattini subgroups. Moreover, the following properties (see Gaschütz [2]) will also be used.

P1. If N is a normal subgroup of a group G and $N \leqq \Phi(U)$, for a subgroup U of G, then $N \leqq \Phi(G)$.

P2. If N is a normal subgroup of a group G and T is the normalizer in G of a Sylow p-subgroup P of N, then $G = NT$.

P3. If $N \leqq \Phi(G)$ is a normal subgroup of G, then $\Phi(G/N) = \Phi(G)/N$.

All groups will be assumed finite.

2. Pseudo-Frattini subgroups. A maximal subgroup of a group is either normal or self-normalizing, i.e., a subgroup which coincides with its normalizer. Moreover, the two classes remain invariant under an automorphism of the group.

DEFINITION 2.1. For a group G denote by $R(G)$ the intersection of the normal maximal subgroups (defining $R(G) = G$ if no normal maximal subgroups exist), and denote by $L(G)$ the intersection of the self-normalizing maximal subgroups (defining $L(G) = G$ if no self-normalizing maximal subgroups exist).

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