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ON ANTI-COMMUTATIVE ALGEBRAS AND GENERAL LIE TRIPLE SYSTEMS

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A general Lie triple system as defined by K. Yamaguti, is considered as an anti-commutative algebra A with a trilinear operation $[x, y, z]$ in which (among other things) the mappings $D(x, y): z \to [x, y z]$ are derivations of A. It is shown that if the trilinear operation is homogeneous, and A is irreducible as a general L. t. s. or irreducible relative to the Lie algebra $I(A)$ generated by the $D(x, y)$'s, then A is a simple algebra. The main result is the following. If A is a simple finite-dimensional anti-commutative algebra over a field of characteristic zero which is a general L. t. s. with a homogeneous trilinear operation $[x, y, z]$, then A is (1) a Lie algebra; or (2) a Malcev algebra; or (3) an algebra satisfying $J(x, y, z)w = J(w, x, yz) + J(w, y, zx) + J(w, z, xy)$ where $J(x, y, z) = xy\cdot z + yz\cdot x + zx\cdot y$. Furthermore in all three cases $I(A)$ is the derivation algebra of A and $I(A)$ is completely reducible in A.

1. *A general Lie triple system* (*general* L. t. s.) has been defined in [6] to be a vector space V over a field F which is closed with respect to a trilinear operation $[x, y, z]$ and a bilinear operation xy so that

$$[x, y, z] = 0, \tag{1.1}$$

$$x^2 = 0, \tag{1.2}$$

$$[x, y, z] + [y, z, x] + [z, x, y] - (xy)z - (yz)x - (zx)y = 0, \tag{1.3}$$

$$[wx, y, z] + [xy, w, z] + [yw, x, z] = 0, \tag{1.4}$$

$$[[u, v, w], x, y] + [[v, u, x], w, y] + [v, u, [w, x, y]] + [w, x, [u, v, y]] = 0, \tag{1.5}$$

$$[w, x, yz] + z[w, x, y] + y[x, w, z] = 0. \tag{1.6}$$

A general L. t. s. is an extension of a Lie triple system used in differential geometry and Jordan algebras. Next we note that if V is a Lie algebra with multiplication xy, then V becomes a general L. t. s. by setting $[x, y, z] = (xy)z$. As an extension of this it was shown in [7] that if V is a Malcev algebra [2] with multiplication xy, then V becomes a general L. t. s. by setting $[x, y, z] = -(xy)z + (yz)x + (zx)y$.

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