PACIFIC JOURNAL OF MATHEMATICS
Vol. 15, No. 2, 1965

CHANGING SIGNS OF FOURIER COEFFICIENTS

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Beginning with a mild extension of a theorem of Littlewood, as generalised by Helgason and by Grothendieck from the circle to a general compact Abelian group G, we derive some properties of the Fourier series of continuous functions on G in relation to arbitrary changes of sign of the coefficients. The main result of this latter type sharpens a fact known for the circle by showing that a continuous function f on G and a ± 1-valued function ω on the character group X may be chosen so that

$$T_\omega f = \Sigma_{\xi \in X} \omega(\xi) \hat{f}(\xi) \xi$$

belongs to no Orlicz space $L_A(G)$ for which $\lim_{u \to \infty} u^{-2} A(u) = \infty$. Similar results are obtained which apply when f is assumed to be merely integrable: in this case one can assert little more than that $T_\omega f$ is a pseudomeasure on G.

NOTATION. With the sole exception of (3.5), G denotes a compact Abelian group and X its character group. We write Ω for the set of all functions on X taking only the values ± 1, and Ω^* for the set of all complex-valued functions on X having absolute value everywhere equal to 1.

The symbol $L^p(G)$ denotes the usual Lebesgue space formed with the Haar measure on G, and likewise for $l^p(X)$ and the (purely discontinuous) Haar measure on X. $M(G)$ is the space of complex Radon measures on G, and $C(G)$ the space of complex-valued continuous functions on G with the usual sup norm.

1. A Littlewood-type theorem.

(1.1) THEOREM. *If F is a complex-valued function on X with the property that for each $\omega \in \Omega$ the series*

(1.1.1) $$\Sigma \omega(\xi) F(\xi) \xi$$

is a Fourier-Stieltjes series, then $F \in l^2(X)$.

(1.2) REMARKS. If G is the circle group, and if the Fourier series are taken in their "real" form, the stated conclusion is known to follow from the hypothesis that each series (1.1.1) is a Fourier series: see [7], p. 215, where a proof is based upon the properties of Rademacher series. One difficulty attending an extension of this

Received April 27, 1964.