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ON RELATIVE COIMMUNITY

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The paper relates to questions raised by A. A. Muchnik in a 1956 Doklady abstract, namely, whether a noncreative r.e. set can be simple in a creative one, and whether a creative r.e. set can be simple in a noncreative one. We furnish a negative answer to the second question, and give a variety of partial results having to do with the first. Thus, we show that no universal set can have immune relative complement inside a noncreative r.e. set and that any r.e. set which is hyperhypersimple in a creative set must itself be creative; whereas, there exist three sets α, β, γ, $\alpha \subseteqq \beta \subseteqq \gamma$, such that β is creative, α and γ are nonuniversal, and both $\beta - \alpha$ and $\gamma - \beta$ are hyperhyperimmune.

In addition, we answer two questions of J. P. Cleave regarding the comparison of effectively inseparable (e.i.) and "almost effectively inseparable" (almost e.i.) sequences of r.e. sets. Thus: a sequence can be almost e.i. without being e.i.; and an almost e.i. sequence of disjoint r.e. sets may have a noncreative union.

1. In [7], Muchnik formulated (in slightly different language) the following two problems: given two r.e. sets Δ, Σ, with $\Delta \subseteqq \Sigma$ and $\Sigma - \Delta$ immune, can we have

(1) Δ creative and Σ mesoic?

(2) Δ mesoic and Σ creative?

In the present paper, we consider these questions relative to not-necessarily-r.e. universal sets; and we make two or three applications of our results to matters considered in [7] and [1]. We are indebted to J. P. Cleave for providing us with a draft copy of [1], which has since been supplanted by a (forthcoming) joint paper of Cleave and C. E. M. Yates. (For an abstract of the Cleave-Yates paper, see [2].)

2. **Definitions and preliminary lemmas.** Basic terminology is essentially as in [3]. Notational departures from [3]: we use 'W_x' in place of 'ω_x', 'ϕ', in place of '0' for the null set, '$\cup$' for union, '$\cap$' for intersection, and '$-$' instead of a prime symbol for complementation. A set Δ of natural numbers is said to be *immune* just in case Δ is infinite and, for all i, if $W_i \subseteqq \Delta$ then W_i is finite. If Δ,

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