PACIFIC JOURNAL OF MATHEMATICS
Vol. 15, No. 4, 1965

THE UNIFORMIZING FUNCTION FOR CERTAIN SIMPLY CONNECTED RIEMANN SURFACES

HOWARD B. CURTIS, JR.

This paper contains a definition of a class of simply connected Riemann surfaces, the determination of the type of a surface from this class, and a representation of the uniformizing function and its derivative as infinite products of quotients as well as quotients of infinite products.

Definition of the class of surfaces. Let $\{a_{2n-1}\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ be two sequences of real numbers such that for $n \geqq 1$,

$$0 < a_{2n-1} < b_{2n-1} < b_{2n}$$

and $b_{2n+1} < b_{2n}$. A surface F of the class to be discussed consists of sheets S_n, $n = 1, 2, 3, \cdots$, over the w-sphere, where for S_n a copy of the w-sphere,

(a) S_1 is slit along the real axis from a_1 to b_1.

(b) For $n \geqq 1$, S_{2n} is slit along the real axis from a_{2n-1} to b_{2n-1} and from b_{2n} to $+\infty$.

(c) For $n \geqq 1$, S_{2n+1} is slit along the real axis from a_{2n+1} to b_{2n+1} and from b_{2n} to $+\infty$.

(d) For $n \geqq 1$, S_n is joined to S_{n+1} along the slits to make the b_n coincide and to form first order branch points at the endpoints of the slits.

The uniformizing function. Because F is simply connected and noncompact, there exists a unique function g which maps F schlichtly and conformally onto $\{|z| < R \leqq \infty\}$, where for $f(z) = g^{-1}(z)$, $f(0) = 0 \in S_1$ and $f'(0) = 1$. Two surfaces of hyperbolic type are obtained by slitting each sheet of F along the uncut parts of the real axis, and an application of the reflection principle to the uniformizing function of one of these surfaces shows that $f(z)$ is real for real z. Let $f(\alpha_{2k-1}) = a_{2k-1}$, $f(-\beta_k) = b_k$, $f(\gamma_{2k}) = \infty \in S_{2k}$ and S_{2k+1}, $f(-\gamma_1) = \infty \in S_1$, and $f(\delta_k) = 0 \in S_k$. The image of F in the z-plane satisfies the following properties. The image of S_n is a region which is symmetric about the real axis. S_1 is mapped onto a domain containing the origin and bounded by a simple closed curve C_1 which intersects the real axis at $-\beta_1$ and α_1. For $n \geqq 2$, S_n is mapped onto an annular region about the origin and bounded by two simple closed curves C_{n-1} and C_n, which

Received July 24, 1964. Research for this paper was supported by a grant from the University of Texas Research Institute and revision of the paper was completed while the author was a Research Fellow in Mathematics at Rice University.