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COMMUTATIVE F-ALGEBRAS

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We extend several theorems for commutative Banach algebras to topological algebras with a sequence of semi-norms (F-algebras). The question of what functions "operate" on an F-algebra is considered. It is proven that analytic functions in several complex variables operate by applying a theorem due to Waelbroeck. If all continuous functions operate on an F-algebra, then it is an algebra of continuous functions. However, unlike the situation for Banach algebras [6], it is not true that if $\sqrt{\ }$ operates the algebra is $C(\Delta)$. This will be shown by an example. A theorem due to Curtis [4], concerning continuity of derivations when the algebra is regular is extended to F-algebras. The result is applied to an algebra of Lipschitz functions to show that it has only a trivial derivation.

Preliminaries. Throughout this paper the letter A will stand for a commutative F-algebra. An F-algebra is a topological algebra with topology determined by a sequence of algebraic semi-norms. The nth semi-norm of an element x in A will be written $\| x \|_n$. We may and shall always assume that for all x in A, $\| x \|_n \leqq \| x \|_{n+1}$. Δ^+ will denote the topological space of all continuous multiplicative linear functionals on A with the weak* topology. Δ will denote Δ^+ minus the zero functional with the relativized topology. For x in A, $\hat{x}$ will be the function in $C(\Delta^+)$ (the continuous functions on Δ^+ with the compact-open topology) defined by $\hat{x}(\varphi) = \varphi(x)$. A will be called regular if given φ_0 in Δ and V a neighborhood of φ_0, there is an element x in A such that $\varphi_0(x) = 1$ and $\varphi(x) = 0$ for $\varphi \notin V$. A will be called semi-simple if $\hat{x} = 0$ implies $x = 0$.

A basic device in the study of F-algebras is to represent A as the inverse limit of a sequence of Banach algebras $\{A_n\}$ where A_n is the completion of A/I_n with norm $\| x + I_n \| = \| x \|_n$ and I_n is the ideal of all x in A such that $\| x \|_n = 0$. The homomorphism $\pi_{m,n}$: $A_n \rightarrow A_m$ for $m \leqq n$ is defined as the completion of the mapping $x + I_n \rightarrow x + I_m$. This representation enables one to construct an element in A by constructing a sequence $\{x_n\}$ such that for each n,

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