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A DESCRIPTION OF MULT$_i$ $(A^1, \cdots, A^n)$ BY GENERATORS AND RELATIONS

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If R is a ring (with unit) and $A^1_R, {}_RA^2_R, \cdots, {}_RA^{n-1}_{R,R}A^n$ are R-(bi)modules, then $\mathrm{Mult}^{R,n}_i(A^1, \cdots, A^n)$ is defined to be the ith left derived functor of the multiple tensor product $A^1 \otimes \cdots \otimes A^n$ $(\otimes = \otimes_R)$; i.e., $H_i(K^1 \otimes \cdots \otimes K^n)$, where each K^r is a projective resolution of A^r.

The purpose of this paper is to give a description of $\mathrm{Mult}^{R,n}_i(A^1, \cdots, A^n)$ in terms of generators and relations, analogous to that given by MacLane in the case $n=2$ [and $\mathrm{Mult}_i = \mathrm{Tor}^R_i(A^1, A^2)$].

Throughout this paper R is a ring with unit, all modules are unitary, and $\otimes$ means $\otimes_R$. If $A^1_R, {}_RA^2_R, \cdots, {}_RA^{n-1}_R, {}_RA^n$ are R-modules (or bimodules, as indicated), then

$$\mathrm{Mult}^{R,n}_i(A^1, \cdots, A^n)$$

is defined to be the ith left derived functor of the multiple tensor product $A^1 \otimes \cdots \otimes A^n$; i.e.

$$H_i(K^1 \otimes \cdots \otimes K^n) ,$$

where each K^r is a projective resolution of A^r. When no confusion can arise we shall often write Mult_i or Mult^n_i in place of $\mathrm{Mult}^{R,n}_i$. Note that for $n = 2$, Mult_i is simply the functor $\mathrm{Tor}^R_i(A^1, A^2)$.

A description of $\mathrm{Mult}^{Z,n}_i$ $(A^1, \cdots, A^n)$ is given in [1]. MacLane [2] has described $\mathrm{Tor}^R_i(A^1, A^2)$ in terms of generators and relations. The purpose of this paper is to extend this description to the functors $\mathrm{Mult}^{R,n}_i(A^1, \cdots, A^n)$. The first difficulty in doing this is to formulate the proper definition of the generators and defining relations. Once this is done, however, most of the proofs are analogous to (though usually considerably more complicated than) the proofs given for $\mathrm{Tor}^R_i(A^1, A^2)$.

A notable exception to this is Theorem 3.1, in which the results for $n = 2$ are used as the first step in an inductive procedure, which is much simpler than a direct proof. Unfortunately, this technique apparently cannot be applied in the proof of the crucial Theorem 3.6, where we must resort to a long and somewhat involved procedure.

Throughout this paper we shall often use the term R-module for left-R-modules, right R-modules, or R-bimodules, the specific meaning being indicated by the context.

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