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PRIMAL CLUSTERS

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In a series of recent publications [Math. Z, 66 (1957), 452–469; Math Z, 62 (1955), 171–188] Foster introduced and studied the theory of a "primal cluster", —a concept which embraces *classes* of algebras of such diverse nature as the classes of all (i) prime-fields, (ii) "n-fields", (iii) basic Post algebras. Here, a primal cluster is essentially a class $\{U_i\}$ of primal (=strictly functionally complete) algebras of the same species such that every finite subset of $\{U_i\}$ is "independent". The concept of independence is essentially a generalization to universal algebras of the Chinese residue Theorem in number theory. Each cluster, $\tilde{U}$, equationally defines —in terms of the identities jointly satisfied by the various finite subset of $\tilde{U}$-a class of "$\tilde{U}$-algebras", and a structure theory for these $\tilde{U}$-algebras was established by Foster, —a theory which contains well known results for Boolean rings, p-rings, and Post algebras. In order to expand the domain of applications of this theory, one should then look for primal clusters. In this paper a permutation, $\frown$, of ths residue class ring R_n, mod n, is constructed, such that $\{(R_n, \times, \frown)\}$ forms a primal cluster. In Theorem 9, which is the main result of this paper, it is shown that a much more comprehensive (and quite "heterogeneous") class K of algebras nevertheless forms a primal cluster. Indeed, K here is the union of all nonisomorphic algebras in the classes of all (i) residue class rings, (ii) basic Post algebras, and (iii) "n-fields". Thus, the primal cluster K furnishes an extension of the primal clusters which were previously given by Foster (loc. cit.).

In a series of recent publications ([1]–[3]) Foster introduced and studied the theory of a "primal cluster", —a concept which embraces *classes* of algebras of such diverse nature as (i) the class of all prime-fields, (ii) the class of all "n-fields", (iii) the class of all basic Post algebras, and (iv) the union of the primal clusters (ii) and (iii) above. Here, a primal cluster is essentially a class $\{U_i\}$ of universal algebras U_i (all of the same species), each is primal (=strictly functionally complete), and such that every finite subset of $\{U_i\}$ is "independent". The concept of independence is essentially a generalization to universal algebras of the Chinese residue Theorem in number theory. Each cluster, $\tilde{U}$, equationally defines—in terms of the identites jointly satisfied by the various finite subsets of $\tilde{U}$-a class of "$\tilde{U}$-algebras", and a structure theory for these $\tilde{U}$-algebras was established in [1],

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