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HARNACK'S INEQUALITIES ON THE CLASSICAL CARTAN DOMAINS

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Recently an extensive work by L. K. Hua on harmonic analysis in Cartan domains, which are called the classical domains, has been translated into English. Here we give Harnack's inequalities for the four main types of Cartan domains treated by Hua.

Harnack's inequality on a type of Cartan domain was obtained [6] for the case of square matrix spaces. Some of these inequalities are application and extension of the results of [6]. I am grateful to Professor J. Mitchell for her encouragement and comments on writing this paper.

Let z be a matrix of complex entries, $z^* = \bar{z}'$ the complex conjugate of the transposed matrix z' and I the identity matrix. Also $H > 0$ means that a hermitian matrix H is positive definite. The first three types of Cartan domains are defined by $D_k = \{z : I - zz^* > 0\}$, $k = 1, 2, 3$, where for $D_1 \equiv D_1(m, n)$, z is an (m, n) matrix (Since the conditions $I - zz^* > 0$ and $I - z^*z > 0$ are equivalent we assume for definiteness that $m \leqq n$.), for $D_2 \equiv D_2(n)$, z is a symmetric matrix of order n and for $D_3 \equiv D_3(n)$, z is a skew-symmetric matrix of order n. The fourth type, $D_4 \equiv D_4(1, n)$, is the set of all $(1, n)$ matrices, or n-dimensional vectors $(n > 2)$, of complex numbers satisfying the conditions

$$(1) \qquad 1 + |zz'|^2 - 2zz^* > 0 , \qquad |zz'| < 1 .$$

It is known that each of the domain D_k possesses a distinguished boundary [1] or characteristic manifold [2, p. 6] $C_k : C_1 \equiv C_1(m, n)$ consists of the (m, n) matrices u satisfying the condition $uu^* = I$. $C_2 \equiv C_2(n)$ consists of all symmetric unitary matrices of order n. $C_3 \equiv C_3(n)$ [2, p. 71] consists of all matrices u of the form $u = w's_1w$, where w is an n-rowed unitary matrix and

$$(2) \qquad s_1 = \begin{cases} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \dotplus \cdots \dotplus \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} & \text{for even } n \\ \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \dotplus \cdots \dotplus \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \dotplus 0 & \text{for odd } n. \end{cases}$$

$C_4 \equiv C_4(1, n)$ consists of all $(1, n)$ matrices u of the form

$$(3) \qquad u = e^{i\theta}x , \qquad xx' = 1 , \qquad 0 \leqq \theta \leqq \pi$$

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