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ON TWO-SIDED H*-ALGEBRAS

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We call a Banach algebra A, whose norm is a Hilbert space norm, a *two-sided H^*-algebra* if for each $x \in A$ there are elements x^l, x^r in A such that $(xy, z) = (y, x^l z)$ and $(yx, z) = (y, zx^r)$ for all $y, z \in A$. A two-sided H^*-algebra is called *discrete* is each right ideal R such that $\{x^r \mid x \in R\} = \{x^l \mid x \in R\}$ contains an idempotent e such that $e^r = e^l = e$. The purpose of this paper is to obtain a structural characterization of those two-sided H^*-algebras M which consist of complex matrices $x = (x_{ij} \mid i, j \in J)$ (J is any index set) for which

$$\sum_{i,j} t_i \mid x_{ij} \mid^2 t_j$$

converges. Here t_i is real and $1 \leqq t_i \leqq a$ for all $i \in J$ and some real a. The inner product in M is

$$(x, y) = \sum_{i,j} t_i x_{ij} \bar{y}_{ij} t_j$$

and

$$x^r_{ij} = (t_i/t_j)\bar{x}_{ji}\,, \qquad x^l_{ij} = (t_j/t_i)\bar{x}_{ji}\,.$$

Then every algebra M is discrete simple and proper ($Mx = 0$ implies $x = 0$). Conversely every discrete simple and proper two-sided H^*-algebra is isomorphic to some algebra M. An incidental result is that the radical of a two-sided H^*-algebra is the right (left) annihilator of the algebra.

In this paper we will refer to such an algebra M above as a *canonical* algebra. We studied two-sided H^*-algebras (and more general algebras) in two previous papers [4, 5]. When $x^r = x^l$ for all x in A we have the H^*-algebras of Ambrose [1] and if we omit x^l we have the right H^*-algebra of Smiley [6]. Incidentally, in [4, Theorem 2] we proved that a proper right H^*-algebra is a two-sided H^*-algebra. So most of the theory of this paper applies to a right H^*-Algebra.

Our proof of the main result (Theorem 4) uses the technique of Ambrose [1] and the lemmas about existence of minimal two-sided projections (Theorem 3 and Lemma 6).

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2. A general theorem.

The following theorem may be of an independent interest (compare with § 2 in [1]).

THEOREM 1. *The radical $\mathfrak{R}$ of each two-sided H^*-algebra A*

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