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A CHARACTERIZATION OF THE GROUP ALGEBRAS OF FINITE GROUPS

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The following is proved:

MAIN THEOREM. Let A be a finite dimensional Archemedian lattice ordered algebra which satisfies the following axioms:

MO If $f, g, h \in A$, and if $f \geqq 0$, then

(l) $f*(g \vee h) = \vee \{f_1*g + f_2*h\colon f_1 \geqq 0, f_2 \geqq 0, f_1 + f_2 = f\}$.

(r) $(g \vee h)*f = \vee \{g*f_1 + h*f_2\colon f_1 \geqq 0, f_2 \geqq 0, f_1 + f_2 = f\}$.

P If $f, g \in A$, and if $f > 0$, $g > 0$, then $f*g > 0$.

Then there exists a finite group G such that A is order and algebra isomorphic to the group algebra of G.

Some similar results are obtained for finite semigroups, and a few applications of these results are given. In particular it is shown that the second cohomology group, $H^2(S, R)$, of any finite commutative semigroup, S, with coefficients in the additive group of real numbers, R, is trivial.

It is well known that two nonisomorphic finite groups can have isomorphic group algebras (over the real or complex numbers) (see e.g. [5, p. 305]). On the other hand, Kawada [4] has shown that if the group algebra is considered as an ordered algebra (with the usual partial ordering obtained by viewing its elements as functions on the group), then if the group algebras of two groups are order as well as algebraically isomorphic, then the two groups are isomorphic (in fact Kawada proved this for the much more general case of locally compact groups). In view of this result it is natural to try to characterize those ordered algebras which occur as the group algebras of finite groups. In this paper we prove the characterization stated above.

All of our results are stated for ordered algebras over the real numbers, but with trivial modifications they apply to ordered algebras over the complex numbers. We will always denote the product of two elements, f and g, of an algebra by $f*g$.

As we will indicate in § 2, a finite dimensional Archemedian lattice ordered vector space is always boundedly lattice complete, so the right hand sides of MOl and MOr always exist.

It follows, of course, from Kawada's result that G in the Main Theorem is unique to within isomorphism.

In § 1 we will give some motivation for axiom MO.

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