PACIFIC JOURNAL OF MATHEMATICS
Vol. 16, No. 2, 1966

# A BRANCHING LAW FOR THE SYMPLECTIC GROUPS

WILLARD MILLER, JR.

**A "branching law" is derived for the irreducible tensor representations of the symplectic groups, and a relation is given between this law and the representation theory of the general linear groups.**

Branching laws for the irreducible tensor representations of the general linear and orthogonal groups are well-known. Furthermore, these laws have a simple form [1]. In the case of the symplectic groups, however, the branching law becomes more complicated and is expressed in terms of a determinant. We derive this result here by brute force applied to the Weyl character formulas, though it could also have been obtained from a more sophisticated treatment of representation theory contained in some unpublished work of Kostant.

**The Branching law.** Let $V^n$ be an $n$-dimensional vector space over the complex field. The symplectic group in $n$ dimensions, $S_p(n/2)$, is the set of all linear transformations $a \in \mathscr{E}(V^n)$, under which a non-degenerate skew-symmetric bilinear form on $V^n \times V^n$ is invariant, [3]. If $\langle \cdot, \cdot \rangle$ is the bilinear form on $V^n \times V^n$ and $a \in \mathscr{E}(V^n)$, then

$$(1) \qquad a \in S_p(n/2) \text{ if and only if } \langle ax, ay \rangle = \langle x, y \rangle \text{ for all } x, y \in V^n .$$

It is well-known that $S_p(n/2)$ can be defined only for even dimensional spaces, ($n = 2\mu$, $\mu$ an integer). It is always possible to choose a basis $e_i, e'_i$, $i = 1, \cdots, \mu$ in $V^n$ such that

$$(2) \qquad \begin{aligned} &\langle e_i, e_j \rangle = \langle e'_i, e'_j \rangle = 0 \quad 1 \leqq i, j \leqq \mu \\ &\langle e_i, e'_j \rangle = \delta_{ij} . \end{aligned}$$

We assume that the matrix realization of $S_p(\mu)$ is given with respect to such a basis [3]. The *unitary symplectic group*, $US_p(\mu)$, is defined by

$$(3) \qquad US_p(\mu) = S_p(\mu) \cap U(2\mu)$$

where $U(2\mu)$ is the group of unitary matrices in $2\mu$ dimensions. The irreducible continuous representations of $US_p(\mu)$ can be denoted by ${}^{\mu}\omega_{f_1,\ldots,f_\mu}$, where $f_1, f_2, \cdots, f_\mu$ are integers such that $f_1 \geqq f_2 \geqq \cdots \geqq f_{\mu-1} \geqq f_\mu \geqq 0$.

Received May 8, 1964. This work represents results obtained in part at the Courant Institute of Mathematical Sciences, New York University, under National Science Foundation grant GP-1669.