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FIRST AND SECOND CATEGORY ABELIAN GROUPS WITH THE n-ADIC TOPOLOGY

EDGAR J. HOWARD

Throughout this paper the word group shall mean Abelian group. The n-adic topology of a group G is formed by taking the subgroups $k!G$ as a base for the neighborhood system of the identity where k is a nonnegative integer. In this paper we list some properties of first and second category groups with the n-adic topology (a group is of first category if it is a countable union of nowhere dense sets).

We characterize first and second category groups and prove the following:

THEOREM: A torsion group is of second category if and only if $G = H \oplus D$ where H is bounded and D is divisible.

THEOREM: Every torsion homomorphic image of a second category (e.g. complete) group is the direct sum of a bounded group and a divisible group.

THEOREM: If G is reduced and of second category and $G = \sum G_\alpha$, then there exists an integer n such that $nG_\alpha = 0$ for all but finitely many α.

THEOREM: If T is torsion, T is isomorphic to the torsion subgroup of a second category group.

The notation and terminology will be essentially that of L. Fuchs in [1]. The topological notations will be those of [6]. We note the following.

1. If A is a subset (subgroup) of B we write $A \subseteqq B$ ($A \leqq B$).
2. $\langle x \rangle$ denotes the cyclic group generated by x.
3. $+$ means sum not necessarily direct, and $\oplus$ means direct sum.
4. By a first (second) category group G we shall mean that G is of first (second) category with the n-adic topology.
5. The term "closed" will be used in the topological sense.

2. On first and second category groups. In this section we shall use the facts that homomorphisms are continuous maps in the n-adic topology, and "onto" homomorphisms are open maps in this topology. The proof of the following is routine.

LEMMA 2.1. *If f is a homomorphism from G onto H, then*

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