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DIFFERENTIABILITY OF SOLUTIONS OF ORDINARY DIFFERENTIAL EQUATIONS IN HILBERT SPACE

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Consider the differential equation

$$\frac{1}{i}\frac{du}{dt} - A(t)u = f(t) \quad (a < t < b) \tag{1.1}$$

where $u(t)$, $f(t)$ are elements of a Hilbert space E and $A(t)$ is a closed linear operator in E with a domain $D(A)$ independent of t and dense in E. Denote by $C^m(a, b)$ the set of functions $v(t)$ with values in E which have m strongly continuous derivatives in (a, b). Introducing the norm

$$|v|_m = \left\{\sum_{j=0}^{m}\int_a^b |v^{(j)}(t)|^2\,dt\right\}^{1/2} \tag{1.2}$$

where $|v(t)|$ is the E-norm of $v(t)$, we denote by $H^m(a, b)$ the completion with respect to the norm (1.2) of the subset of functions in $C^m(a, b)$ whose norm is finite. Set $H^m = H^m(-\infty, \infty)$ and denote by H_0^m the subset of functions in H^m which have compact support. The solutions $u(t)$ of (1.1) are understood in the sense that $u(t) \in H^1(a', b')$ for any $a < a' < b' < b$.

THEOREM 1. **Assume that, for each $a < t < b$, the resolvent $R(\lambda, A(t)) = (\lambda - A(t))^{-1}$ of $A(t)$ exists for all real λ, $|\lambda| \geqq N(t)$, and that**

$$|R(\lambda, A(t))| \leqq \frac{C(t)}{|\lambda|} \text{ if } \lambda \text{ real, } |\lambda| \geqq N(t)\,, \tag{1.3}$$

where $N(t)$, $C(t)$ are constants. Assume next that for each $s \in (a, b)$, $A^{-1}(s)$ exists and

(1.4) $A(t)A^{-1}(s)$ **has m uniformly continuous t-derivatives,**

for $a < t < b$, where m is any integer $\geqq 1$. If u is a solution of (1.1) and if $f \in H^m(a, b)$, then $u \in H^{m+1}(a', b')$ for any $a < a' < b' < b$.

THEOREM 2. **If the assumptions of Theorem 1 hold with $m = \infty$, if $A(t)A^{-1}(s)$ is analytic in $t(a < t < b)$ for each $s \in (a, b)$, and if $f(t)$ is analytic in (a, b), then $u(t)$ is also analytic in (a, b).**

In case E is a Banach space, an analogue of Theorem 1 was proved by Sobolevski [3] and Tanabe [4] and an analogue of Theorem 2 was proved by Sobolevski [3] and Komatzu [2], but all these authors

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