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OPERATORS COMMUTING WITH TRANSLATIONS

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This paper is concerned with the representation, in terms of convolutions with pseudomeasures, of continuous linear operators which commute with translations and which transform continuous functions with compact supports on a Hausdorff locally compact Abelian group G into restricted types of Radon measures on G. The two main theorems each assert that any such operator T is of the form $Tf = s * f$ for a suitably chosen pseudomeasure s on G; the assertions differ in detail in respect of the hypotheses imposed on the range of T. The second theorem is an extension of Proposition 2 of [1] from the case in which G is a finite product of lines and/or circles to the general situation.

Preliminaries. The notations are as described in § 1 of [1], with G in place of X, and with the following additions. If $K \subset G$, $C_K(G)$ denotes the set of $f \in C_c(G)$ satisfying supp $f \subset K$. The symbol $M_b(G)$ will denote the set of all bounded Radon measures on G. Continuity of the operators T considered will, in the absence of any indication to the contrary, refer to the inductive limit topology on $C_c(G)$ and the vague topology $\sigma(M(G), C_c(G))$ on $M(G)$ and its subsets. No distinction is drawn between a locally integrable function f on G and the associated measure $fdx \in M(G)$, dx denoting the element of Haar measure on G. In this paper, X will denote the character group of G, the Haar measure $d\xi$ on X being chosen so that the Fourier transformation is an isometry of $L^2(G)$ onto $L^2(X)$.

Prior to stating the representation theorems, we make some remarks about pseudomeasures on G.

Let $A(G)$ denote the space of functions u on G which are inverse Fourier transforms of functions $v \in L^1(X)$:

$$u(x) = \int_X v(\xi)\xi(x)d\xi \ ;$$

$A(G)$ is a Banach space under the norm

$$\| u \|_A = \int_X | v(\xi) | d\xi \equiv \| v \|_1 \ .$$

By a pseudomeasure on G is meant a continuous linear functional on $A(G)$, and we denote by $P(G)$ the set of pseudomeasures on G. By $\| \cdot \|_P$ is meant the usual norm on $P(G)$ qua dual of $A(G)$. The Fourier transformation can be defined for pseudomeasures s in such a way that

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