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THE DEFICIENCY INDEX OF ORDINARY SELF-ADJOINT DIFFERENTIAL OPERATORS

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This paper is concerned with the computation of the deficiency index of an ordinary self-adjoint differential operator with real coefficients. The operator, L, is supposed defined on $[0, \infty)$ and is regular at the origin. The deficiency index counts the number of L^2 solutions to the equation $Ly = zy$, where z is any nonreal complex number.

The results obtained include as rather special cases almost all of the results known to the author when the order of L is larger than two.

The principal tool used is an asymptotic theorem of N. Levinson.

We are interested in computing the deficiency index of an ordinary self-adjoint differential operator,

$$Ly = (-1)^n \frac{d^n}{dt^n}\left(q_0 \frac{d^n y}{dt^n}\right) + (-1)^{n-1} \frac{d^{n-1}}{dt^{n-1}}\left(q_1 \frac{d^{n-1} y}{dt^{n-1}}\right) + \cdots + q_n y \,, \tag{1.1}$$

defined on the interval $[0, \infty)$, with the coefficients q_k real and measurable. We shall suppose that L is regular at the origin which means that $1/q_0, q_1, \cdots, q_n$ belong to L^1 on every finite interval $[0, T]$.

The number m in the deficiency index (m, m) of the minimal operator L_0 associated with the formal operator (1.1) is the dimension of the linear space of L^2 solutions to any equation

$$Ly = zy \,, \qquad Imz \neq 0 \,. \tag{1.2}$$

As is well known, and easy to show, it is always true that $n \leqq m \leqq 2n$.

In the case where the order of the operator in (1.1) is two, fairly sophisticated tests are now available, [1], [5], which tell when the deficiency index in (1, 1). For an order larger than two very little seems to be known. Some results are due to M.A. Neumark [8] who obtains conditions that the deficiency index shall be either (n, n) or $(n+1, n+1)$. Other results are due to S. A. Orlov [9] and F.A. Neimark [7] who obtain the deficiency index of L_0 when the coefficients q_k in (1.1) are essentially of polynomial growth as $t \to \infty$. These results will appear as rather special cases of the considerations which we shall present in this paper. As a by product we can obtain the result, originally proved by Glasmann [4], that the number m can

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