PACIFIC JOURNAL OF MATHEMATICS
Vol. 16, No. 2, 1966

UNIQUE FACTORIZATION IN POWER SERIES RINGS AND SEMIGROUPS

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In this note a short proof is given for a theorem due originally to Deckard and to Cashwell and Everett. The theorem states that every ring of power series over an integral domain R is a unique factorization domain if and only if every ring of power series over R in a finite set of indeterminates is a unique factorization domain. The proof is based on a study of the structure of the multiplicative semigroups of such rings. Much of the novelty and most of the brevity of this argument may be accounted for by the fact that Dilworth's theorem on the decomposition of partially ordered sets is invoked at a crucial point in the proof.

Suppose R is a ring and that R^+ is the additive group of R. Suppose I is a nonvoid set well ordered by $<$. If N is the additive semigroup of the nonnegative integers and M is the weak product (Chevalley [3]) $\prod_{i\in I}^{w} N$, let $R_I = \prod_{m\in M} R^+$. For f, g in R_I define

$$(fg)(m) = \sum_{k+l=m} f(k)g(l) \;;$$

the resulting ring is called a *ring of power series* over R, and is an integral domain if R is. An f in R_I is a unit if and only if $f(0)$ is a unit of R.

Deckard [5] and Cashwell and Everett [2] proved that R_I is a unique factorization domain for every I if R_J is a unique factorization domain for every finite set J. (Thus it follows, from results of Samuel [8] and Buchsbaum [1], that R_I is a unique factorization domain if R is a regular unique factorization domain.)

It suffices to consider either the multiplicative semigroup S_I of the nonzero members of R_I or the quotient of S_I obtained by identifying associates; factorization in a cancellative semigroup being unique if and only if (Clifford [4]) both

(1) The divisor chain condition holds.

(2) All irreducibles are primes.

This note contains a study of factorization in a class of semigroups which, on specialization, yields a direct and concise proof of the Deckard-Cashwell and Everett theorem.

Extensions of semigroups. With I as above, suppose $\{S_i : i \in I\}$

Received May 8, 1964.