PACIFIC JOURNAL OF MATHEMATICS
Vol. 16, No. 2, 1966

ADJOINT QUASI-DIFFERENTIAL OPERATORS OF EULER TYPE

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This paper treats linear quasi-differential operators of the form

$$L[y] = \sum_{j=0}^{n} p_{0j}y^{(j)} - \left(\sum_{j=0}^{n} p_{1j}y^{(j)} - \left(\cdots - \left(\sum_{j=0}^{n} p_{mj}y^{(j)}\right)' \cdots\right)'\right)',$$

based on an integrable $(m+1)\times(n+1)$ matrix function $[p_{ij}]$, $(i=0,\cdots,m;\ j=0,\cdots,n)$, about which suitable regularity assumptions are made. Results obtained by Reid (Trans. Amer. Math. Soc. Vol. 85 (1957), pp. 446–461) are extended to operators of the type considered here.

A generalized Green's function for the system $\{L[y]=0, y\in\mathscr{D}\}$ is defined, where $\mathscr{D}$ is a linear subspace of the domain of L. Resolvent and deterministic properties of this function are presented, together with the relationship of such a generalized Green's function to the generalized Green's function for the associated adjoint system.

For a large class of two-point boundary problems in which the boundary conditions involve the characteristic parameter linearly it is shown that there exists a simultaneous canonical representation of the boundary conditions for a given problem and those of its adjoint; in particular, in the self-adjoint case this canonical representation has the form of boundary conditions and transversality conditions for a variational problem. Finally, these results are applied to a two-point boundary problem involving a differential operator of the type considered in the paper of Reid above.

Since an important example of an operator of the form of $L[y]$ is the Euler operator in the calculus of variations, we shall refer to such operators as *quasi-differential operators of Euler type*.

Section 2 gives a more precise description of the operator, and Section 3 is concerned with a discussion of its adjoint. In particular it is shown that if $\mathscr{D}_0$ is the class of functions y in the domain of L with the property that the functions $y, y', \cdots, y^{(n-1)}$, $\tilde{y}_m \equiv \sum_{j=0}^{n} p_{mj}y^{(j)}$, $\tilde{y}_i \equiv \sum_{j=0}^{n} p_{ij}y^{(j)} - \tilde{y}'_{i+1}$, $(i = m-1, \cdots, 1)$, vanish at a and at b, and if T_0 is the restriction of L to $\mathscr{D}_0$, then the adjoint operator T_0^* is given by

Received May 21, 1964. The author wishes to express his sincere appreciation to Professor W. T. Reid for his valuable suggestions and assistance throughout the preparation of this Ph. D. dissertation (State University of Iowa, June 1964). The research for this paper was supported by the Air Force Office of Scientific Research, under grants AF-AFOR-62-78 and AF-AFOR-438-63. The authors address is now: Department of Mathematics, University of Tennessee, Knoxville.