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REFLECTION AND APPROXIMATION BY INTERPOLATION ALONG THE BOUNDARY FOR ANALYTIC FUNCTIONS

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Let there be given a function $f(z)$ analytic in an open connected set, not necessarily simply connected, which is bounded by simple closed analytic curves such that the function is continuous on the closure of the region and such that the real part of the function satisfies boundary conditions that are analytic in a neighborhood of the boundary. We want to interpolate $f(z)$ along the boundaries and find conditions that make the interpolants converge maximally to $f(z)$ throughout the closure of the region. The boundary condition on the real part of $f(z)$ permits the analytic continuation of $f(z)$ across the boundary curves and ensures that we are interpolating at points interior to the region of analyticity. In our error estimates (Theorem 1) maximal convergence depends in an essential way on how far we can reflect $f(z)$ and this in turn depends on the boundary values of the real part of $f(z)$ as well as on the geometry of the given region and its analytic boundaries. In Theorems 2 and 3, a simply connected region is considered. Special points of interpolation are given, these depend only on the parametric representation of the boundary curves and not a conformal map. These points are the image points of the Chebyshev polynomials.

Finally an example is given for a multiply connected region.

As is well known [2] Runge's beautiful theorem shows us that there exist certain "equidistributed" points on the analytic curves such that if we interpolate at these points the interpolants converge to the function. However, the proof depends on knowing the conformal map in order to know what the interpolation points are. Here we shall give conditions that do not require knowledge of the conformal map but for convergence depend on how far we can reflect. Along with these, we shall give simple error estimates. Moreover, we shall show that possible interpolation points are the images on the boundary of roots of the Chebyshev polynomials.

The aspects of this paper which are novel are

(i) the use of reflection

(ii) interpolation at boundary points which are gotten directly from the parametric representation of the boundary and do not depend on a conformal map

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