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PSEUDOCOMPACTNESS AND UNIFORM CONTINUITY IN TOPOLOGICAL GROUPS

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This work contains a number of theorems about pseudocompact groups. Our first and most useful theorem allows us to decide whether or not a given (totally bounded) group is pseudocompact on the basis of how the group sits in its Weil completion. A corollary, which permits us to answer a question posed by Irving Glicksberg (Trans. Amer. Math. Soc. 90 (1959), 369–382) is: The product of any set of pseudocompact groups is pseudocompact. Following James Kister (Proc. Amer. Math. Soc. 13 (1962), 37–40) we say that a topological group G has property U provided that each continuous function mapping G into the real line is uniformly continuous. We prove that each pseudocompact group has property U.

Sections 2 and 3 are devoted to solving the following two problems: (a) In order that a group have property U, is it sufficient that each bounded continuous real-valued function on it be uniformly continuous? (b) Must a nondiscrete group with property U be pseudocompact? Theorem 2.8 answers (a) affirmatively. Question (b), the genesis of this paper, was posed by Kister (loc. cit.). For a large class of groups the question has an affirmative answer (see 3.1); but in 3.2 we offer an example (a Lindelöf space) showing that in general the answer is negative.

Much of the content of this paper is summarized by Theorem 4.1, in which we list a number of properties equivalent to pseudocompactness for topological groups. We conclude with an example of a metrizable, non totally bounded Abelian group on which each uniformly continuous real-valued function is bounded.

Conventions and definitions. All topological groups considered here are assumed to be Hausdorff. The algebraic structure of the groups we consider is virtually immaterial; in particular, our groups are permitted to be non-Abelian.

A topological group G is said to be totally bounded if, for each neighborhood U of the identity, a finite number of translates of U covers G. It has been shown in [10] by Weil that each totally bounded group is a dense topological subgroup of a compact group and that this compactification is unique to within a topological isomorphism leaving G fixed pointwise. We refer to this compactification of G as the Weil

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