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ON ABSOLUTELY CONTINUOUS FUNCTIONS AND THE WELL-BOUNDED OPERATOR

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The author considers an operator T in a reflexive Banach space X for which there is a bounded operational calculus $a \to a(T)$ defined on $AC(I)$, the algebra of absolutely continuous functions defined on $I = [0, 1]$ with the norm $|a(0)| + \mathrm{Var}_I(a)$ for $a \in AC(I)$. Such operators, called well-bounded, have been investigated by Smart and Ringrose (J. Australian Math. Soc. 1 (1960), 319–343 and Proc. London Math. Soc. (3) 13 (1963), 613–638). The present paper explores a new method for obtaining the spectral theorem for this operator. Let AC_0 be the maximal ideal of members of $AC(I)$ which are zero at 0. The method consists in introducing Arens multiplication into AC_0^{**}, the second conjugate space of AC_0, and in investigating the larger algebra for a suitable family of idempotents which will serve as candidates for bounded spectral projections associated with T. Idempotents in AC_0^{**} are mapped into these projections by means of a homomorphism extension technique which extends the original operational calculus of AC_0 into $B(X)$ (the bounded linear operators on X), to a bounded homomorphism of AC_0^{**} into $B(X)$. The extended homomorphism is defined on a quotient algebra of AC_0^{**}. This quotient algebra turns out to be a copy of all functions of bounded variation on I which are zero at 0 under the usual pointwise operations.

Let $AC(I)$ be the complex algebra of complex-valued, absolutely continuous functions on $I = [0, 1]$ with the algebraic operations being the usual addition and multiplication of functions. This algebra is a Banach algebra under the norm (see Section 3)

$$(1.0.1) \qquad \|a\| = |a(0)| + \operatorname*{Var}_I (a) , \qquad a \in AC(I) .$$

We shall consider a linear operator T in a reflexive Banach space X for which there is an operational calculus $a \longrightarrow a(T)$ satisfying

$$\|a(T)\| \leqq K\|a\| , \qquad a \in AC(I) .$$

This operator, an example of a well-bounded operator, was introduced by Smart [14]. Smart showed that T determines a bounded, strongly

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