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BOUNDARY VALUE PROBLEMS FOR NONLINEAR ORDINARY DIFFERENTIAL EQUATIONS

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Conditions are given under which a quasi-linear differential equation has at least one solution in a given compact interval that satisfies a given system of homogeneous or nonhomogeneous linear constraints. These conditions are not formulated in the space in which the solutions take their values, as is usually done; instead they involve the set of continuous mappings subject to the constraints and the set of forcing terms for which the associated nonhomogeneous linear differential equation has solutions satisfying the constraints. The latter set is, under mild conditions, a topological direct summand of the space of continuous mappings. This occurs in the problem of the existence of periodic solutions which is discussed in detail as illustration.

In the present note we derive simple sufficient conditions in order that a differential equation

$$x' = A(t)x + f(t, x) \tag{1}$$

have at least one solution u in a compact interval K which satisfies a system of constraints of the form

$$c_i(u) = \eta_i\,, \qquad 1 \leqq i \leqq m\,. \tag{2}$$

Here (c_i) is a linearly independent family of continuous linear forms on the Banach space C of continuous mappings of K into X, the underlying real Banach space, and $y = (\eta_i)$ is an arbitrary point in R^m.

Our results are in the spirit of two very general theorems, essentially due to Corduneanu [7], which Hartman and Onuchic [9] have applied to the asymptotic integration of differential equations such as (1). However, unlike these theorems, our considerations do not depend upon the work of Massera and Schäffer (see, e.g., [11], [12]) but instead are based on some elementary facts concerning linear differential equations that admit solutions for which (2) holds. An entirely different treatment of boundary value problems has recently been given by Conti [4], [5], [6].

For the sake of simplicity, we assume throughout that $t \to A(t)$ is a continuous mapping of K into the normed space of continuous endomorphisms of X and that f is a continuous mapping of $K \times K$

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