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GROUPS WHOSE IRREDUCIBLE REPRESENTATIONS HAVE DEGREES DIVIDING p^2

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In several previous papers I. M. Isaacs and this author studied properties of groups which are related to the degrees of their absolutely irreducible representations and in particular to the biggest such degree. The results were concerned mainly with the existence of "large" abelian subgroups in these groups. It was found that much more could be said in the p-group-like situation in which the degrees of the irreducible characters of group G are all powers of a fixed prime p. We say group G has r.x.e (representation exponent e) if the degrees of all the irreducible characters of G divide p^e. In this paper we characterize groups with r.x.2. It is found that the prime $p = 2$ plays a special role here. This supports the conjecture that additional and more complicated groups with r.x.e occur for $p \leqq e$. With a few exceptions for $p = 2$, all groups G with r.x.2 are shown to have either a normal subgroup of index p with r.x.1 or a center of index dividing p^6.

1. Preliminary remarks. We will use here the notation and many of the results of the first four sections of [5]. For example, we need the characterization of groups with r.x.1 given there. From this we obtain the following.

(1.1) LEMMA. (i) *Let N have r.x.1. Then either N has a characteristic abelian subgroup of index p or $[N: \mathfrak{Z}(N)]$ divides p^3 where $\mathfrak{Z}(N)$ denotes the center of N.*

(ii) *Let N be a normal subgroup of G. Suppose G has r.x.e, N has r.x.1 and $[G: N] = p^n$. Then G has a normal abelian subgroup A with $[G: A]$ dividing p^{n+2}.*

Proof. We consider (i) first. By Theorem C of [5] we can assume that N has a normal abelian subgroup A of index p. If A is not characteristic then N has another such subgroup B. Thus $N = AB$, and since both A and B are abelian, $A \cap B \subseteqq \mathfrak{Z}(N)$. Since $[N: A \cap B] = p^2$, this results follows.

If N in (ii) has a characteristic abelian subgroup A of index dividing p^2, the result is clear. Otherwise by (i), $[N: \mathfrak{Z}(N)] < p^3$. Now $\mathfrak{Z}(N) \Delta G$ and $G/\mathfrak{Z}(N)$ is a p-group. Let A be the inverse image in G of a central subgroup of order p of this quotient. Then clearly A is

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